

# Techniques for Received Signal Focusing in DSUWB Systems

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**Abstract**—Several techniques for received signal focusing based on multiple transmit antennas and time-reversal prefiltering (TRP) for direct-sequence ultra-wideband (DSUWB) systems are studied in this work. The use of multiple transmit antennas offers a higher spatial diversity gain for better signal focusing. To perform TRP, we need to feed the channel information back to the transmitter, which may contain hundreds of taps and could be expensive in transmission. The previously proposed partial-Pre-RAKE (PPR) scheme is analyzed and shown to provide the maximum average power concentration if a fixed number of consecutive taps available from the feedback. To further enhance this algorithm, we propose a phase-assisted tap selection scheme. The performance of the proposed phase-assisted tap selection scheme is analyzed and verified by computer simulation.

## I. INTRODUCTION

The ultra-wideband (UWB) radio pulse occupies a huge frequency bandwidth due to its tiny pulse width in the time domain. This also results in an extraordinary multipath resolution which provides an effective means to combat channel fading. It was shown in [1] that at least 20 correlation operations are required in a receiver to catch about one half of the energy spread in the entire channel impulse response (CIR). However, a receiver with so many correlation operations is computationally complex and a significant amount of power would be consumed. As a result, this solution is not suitable for a receiver in a mobile environment.

To address this problem, the time-reversal prefiltering (TRP) technique borrowed from under-water acoustic signal processing was proposed by Stohmer *et al.* [2]. They applied TRP to single-input single-output (SISO) UWB systems by sending the complete CIR from the receiver back to the transmitter. It is shown in [2] that the TR-UWB solution reduces the number of correlation operators and provides a higher data rate and lower interference to other communication systems. A similar idea, known as the Pre-RAKE technique, was proposed in [3] for the code-division multiple access (CDMA) systems. Unlike typical CDMA channels, an UWB channel contains hundreds of channel taps so that feeding the entire CIR back to the transmitter is impractical.

Two methods that deliver only partial CIR back to the transmitter, known as the selective pre-RAKE (SPR) and the partial pre-RAKE (PPR) schemes, were proposed in [5] to reduce the feedback burden for a SISO UWB channel. Since an UWB channel contains many small taps, the SPR scheme passes  $\bar{L}$  strongest taps back to the transmitter. However, the re-sorting

of the whole taps could be complex when the multipath resolution improves. The PPR scheme eliminates the sorting operation by directly sending the first  $\bar{L}$  taps to the transmitter. Since the significant taps may not appear in the beginning of the CIR, PPR renders worse performance than SPR. Even though it was shown in [5] that these two schemes alleviate the feedback burden of the entire CIR, performance analysis is missing due to lack of a proper channel model.

In this work, we generalize the TRP scheme to downlink direct-sequence UWB (DSUWB) systems with multiple transmit antennas. Based on the channel model provided by Chao and Scholtz [4], we analyze the performance of the PPR scheme and show that it results in the highest average power if only consecutive  $\bar{L}$  taps are available at the transmitter. On one hand, the use of multiple transmit antennas provides more spatial diversity so as to concentrate more energy at the peak of the resultant channel. On the other hand, more information is required at the transmitter since more transmit antennas are deployed. Here, we propose a new scheme, called the phase-assisted channel feedback, which separately transmits the phase and the amplitude information of CIR back to the transmitter. When the phase information is available at both sides, the burden for the amplitude information feedback is greatly reduced with the help of the known phase as the side information. Since only one bit is used to represent each phase, the overhead for transmitting the complete phase information is low. The performance of the proposed method is theoretically analyzed and verified by computer simulation.

## II. SYSTEM MODEL

Consider a downlink DSUWB system from an access point (AP) to  $N_u$  mobile devices. The AP has  $N_t$  transmit antennas to acquire spatial diversity while each mobile device has a single receive antenna. Every transceiver pair is assumed to experience uncorrelated fading, and the channel model proposed in [4] is adopted here. Then, the channel between the  $n$ th transmit antenna and device  $k$  can be represented by

$$h_{k,n}(t) = \sum_{i=0}^{L-1} \rho_{k,n}(i) \alpha_{k,n}(i) \delta(t - i\Delta), \quad (1)$$

where  $L$  is the total number of paths,  $\Delta$  is the multipath resolution that is assumed to be the same as the time domain pulse width,  $\rho_{k,n}(i) \in \{+1, -1\}$  with equal probability represents

the phase of the  $i$ th path and the corresponding amplitude,  $\alpha_{k,n}(i)$ , is an independent Rayleigh random variable with the following probability density function (pdf)

$$f_{\alpha_{k,n}(i)}(x) = \frac{x}{\sigma_i^2} e^{-x^2/2\sigma_i^2}. \quad (2)$$

Also, the average power of  $\alpha_{k,n}(i)$ , which is equal to  $2\sigma_i^2$ , is subject to exponential decay, *i.e.*

$$2\sigma_i^2 = \Omega\gamma^i, \quad (3)$$

where  $\Omega$  is the average power of  $\alpha_{k,n}(0)$ ,  $\gamma = e^{-\Delta/\Gamma}$ , and  $\Gamma$  is the decay time constant. Furthermore, the channel is fixed during one package of symbols and changes independently from package to package.

Since UWB radio has an extremely fine multipath resolution, the number of taps involved could be large. The feedback burden would be high if the complete channel information is sent back to the transmitter as suggested in [2]. Instead of feeding all channel taps back to the transmitter, a partial set of channel taps can be selected and sent back to the transmitter for time-reversal prefiltering and the tap selection algorithms will be discussed in the next section. The time-reversal prefilter for  $h_{k,n}(t)$ ,  $p_{k,n}(t)$ , can be described as

$$p_{k,n}(t) = \sum_{s=1}^{L_{k,n}^s} \bar{\alpha}_{k,n}(i_{s,k,n}) \delta(-t - i_{s,k,n}\Delta + \tau_{k,n}^{max}) \quad (4)$$

where  $L_{k,n}^s$  is the total number of selected taps and  $\tau_{k,n}^{max} = \max_s i_{s,k,n}\Delta$  is the minimum delay used to *causalize*  $p_{k,n}(t)$  and

$$\bar{\alpha}_{k,n}(i_{s,k,n}) = \frac{\rho_{k,n}(i_{s,k,n})\alpha_{k,n}(i_{s,k,n})}{\sqrt{\chi_k}}. \quad (5)$$

To keep the transmit power constant, we normalize the total output power and demand

$$\chi_k = \sum_{n=1}^{N_t} \sum_{s=1}^{L_{k,n}^s} \alpha_{k,n}(i_{s,k,n})^2.$$

Thus, the resultant CIR between device  $k$  and the  $n$ th transmit antenna after prefiltering can be written as

$$\hat{h}_{k,n}(t) = h_{k,n}(t) \otimes p_{k,n}(t) = \sum_{l=1}^{\hat{L}_{k,n}} \hat{\alpha}_{k,n}(l) \delta(t - \hat{\tau}_{k,n}(l)), \quad (6)$$

where  $\hat{\alpha}_{k,n}(l)$  and  $\hat{\tau}_{k,n}(l)$  are the amplitude and delay for the  $l$ th tap, respectively, and " $\otimes$ " is the convolution operator. It is expected that

$$\max_t \hat{h}_{k,n}(t) = \hat{h}_{k,n}(\tau_{k,n}^{max}) = \frac{1}{\sqrt{\chi_k}} \sum_{s=1}^{L_{k,n}^s} \alpha_{k,n}(i_{s,k,n})^2. \quad (7)$$

Since the CIR between each transceiver pair is independent, signals coming from different transmitters may be focused at different time. To avoid potential signal collision, the prefiltering sequence  $p_{k,n}(t)$  is zero padded so that signals of the same

user from different antennas are focused at the same time instance. Let  $\hat{h}_{k,n}(t, d_{k,n})$  be the CIR after  $d_{k,n}$  zeros are padded. Then, the overall channel response for device  $k$  can be written as

$$\hat{h}_k(t) = \sum_{n=1}^{N_t} \hat{h}_{k,n}(t, d_{k,n}) = \sum_{i=1}^{\hat{L}_k} \hat{\alpha}_k(i) \delta(t - \hat{\tau}_k(i)). \quad (8)$$

As given above, we may view the proposed signal focusing system as a SISO system.

At the AP, the transmit signal of device  $k$  is spread by a unique  $N$ -chip spreading code,  $[c_k(0), \dots, c_k(N-1)]$ , where  $c_k(i) \in \{+1/\sqrt{N}, -1/\sqrt{N}\}$ ,  $i \leq i \leq N-1$ . If all  $N_u$  signals are transmitted synchronously, the transmit signal can be represented by the following equation,

$$y_s(t) = \sum_{k=1}^{N_u} \sqrt{E_k} \sum_{i=-\infty}^{\infty} b_k(i) \sum_{j=0}^{N-1} c_k(j) w_s(t - j\Delta - iT_s), \quad (9)$$

where  $E_k$  and  $b_k(i)$  are, respectively, the transmit power and the  $i$ th bipolar signal for device  $k$ ,  $w_s(t)$  is the UWB pulse waveform of the unit energy, and  $T_s$  is the symbol interval properly chosen to reduce inter-symbol interference.

With the channel response given in (8), the received signal at device  $k$  can be written as

$$r_k(t) = \hat{h}_k(t) \otimes y_r(t) + n(t), \quad (10)$$

where  $n(t)$  is additive white Gaussian noise (AWGN) of zero mean and with two-sided power spectral density  $\frac{N_o}{2}$ ,

$$y_r(t) = \sum_{k=1}^{N_u} \sqrt{E_k} \sum_{i=-\infty}^{\infty} b_k(i) \sum_{j=0}^{N-1} c_k(j) w_r(t - j\Delta - iT_s), \quad (11)$$

where  $w_r(t)$  is the received UWB pulse waveform, which is different from  $w_s(t)$  due to the antenna effect yet assumed to be known to the receiver.

Without loss of generality, we choose  $k = 1$  to be the target device and  $b_1(i)$  be the desired signal. After chip-matched filtering and sampling, the discrete received signal of device 1 for the  $i$ th transmit symbol can be expressed as

$$\mathbf{r}_1(i) = \sum_{q=1}^{\hat{L}_1} \sqrt{E_1} \hat{\alpha}_1(q) \mathbf{s}_1^{(q)} b_1(i) + \mathbf{I}_1(i) + \mathbf{n}(i), \quad (12)$$

where  $\mathbf{r}_1(i)$  is a  $g \times 1$  vector with  $g = (\max_j \hat{\tau}_1(j)/\Delta) + N - 1$ ,  $\mathbf{s}_1^{(q)}$  is a sparse vector that contains the spreading code of device 1 from the  $(\hat{\tau}_1(q)/\Delta)$ th to the  $(\hat{\tau}_1(q)/\Delta + N - 1)$ th chips and zeros elsewhere,  $\mathbf{I}_1(i)$  is the interference vector that contains multiple-access interference (MAI) and possible inter-symbol interference (ISI), and  $\mathbf{n}(i)$  is the AWGN noise vector.

To decode  $b_1(i)$ , an  $f$ -finger RAKE receiver that maximally combines all finger outputs is deployed. The estimation can be done as

$$\hat{b}_1(i) = \text{sign}(\mathbf{w}_1^T \mathbf{F}_1^T \mathbf{r}_1(i)), \quad (13)$$

where  $\mathbf{F}_1 = [\mathbf{s}_1^{(q_1)}, \dots, \mathbf{s}_1^{(q_f)}]$ ,  $\mathbf{w}_1 = [\hat{\alpha}_1(q_1), \dots, \hat{\alpha}_1(q_f)]^T$  and their superscript  $T$  denotes the transpose operation.

### III. TAP SELECTION ALGORITHMS

Due to the extraordinary multipath resolution, each UWB channel may contain hundreds of taps. Moreover, to minimize the quantization error, more bits are required to represent  $\alpha_{k,n}(i)$ . Since it is difficult to send all the channel information back to the transmitter, a selection algorithm that reduces the amount of feedback information by picking up those important taps with small performance degradation is desired.

Two tap selection algorithms, *i.e.*, the SPR (selective pre-RAKE) and PPR (partial pre-RAKE) schemes, were proposed in [5] to reduce the feedback information amount. The SPR scheme picks up the  $\bar{L}$  strongest CIR taps and demands additional sorting process. The complexity of the sorting process is high due to the high multipath resolution. The PPR scheme feeds only the first  $\bar{L}$  taps back and no sorting is needed. After a brief review of the PPR scheme, under the channel model in [4], we show that the PPR system offers the greatest average power when the number of consecutive feedback taps is bounded by  $\bar{L}$  in Sec. III-A. Then, a phase-assisted channel feedback scheme that transmits the phase and amplitude information back to the transmitter separately is proposed in Sec. III-B to improve the performance of the PPR system.

#### A. PPR Scheme and Its Performance Analysis

The PPR system sends only the first  $\bar{L}$  taps, *i.e.*,  $\{\rho_{k,n}(0) * \alpha_{k,n}(0), \dots, \rho_{k,n}(\bar{L}-1) \alpha_{k,n}(\bar{L}-1)\}$ , back to the transmitter without channel tap sorting [5]. It is straightforward to show that the peak power of the received signal by feeding back the first  $\bar{L}$  taps is

$$P_{peak}^{(p)}(\bar{L}) = \sum_{n=1}^{N_t} \sum_{i=0}^{\bar{L}-1} \alpha_{k,n}(i)^2. \quad (14)$$

Thus, the average power concentrated at the peak is

$$\begin{aligned} \bar{P}_{peak}^{(p)}(\bar{L}) &= E\left\{\sum_{n=1}^{N_t} \sum_{i=0}^{\bar{L}-1} \alpha_{k,n}(i)^2\right\} \\ &= \sum_{n=1}^{N_t} \sum_{i=0}^{\bar{L}-1} \Omega \gamma^i = N_t \Omega \frac{1 - \gamma^{\bar{L}}}{1 - \gamma}, \end{aligned} \quad (15)$$

where " $E\{\}$ " is the expectation operator. We have the following proposition to characterize the performance of the PPR scheme.

**Proposition 1.** The PPR scheme maximizes the average peak power for a fixed number of consecutive taps available, and the average power concentrated at the peak is equal to  $N_t$  times the sum of the  $\bar{L}$  largest eigenvalue of matrix  $\mathbf{R}_{\mathbf{h}_{k,n}} \equiv E\{\mathbf{h}_{k,n} \mathbf{h}_{k,n}^T\}$ , where  $\mathbf{h}_{k,n} = [h_{k,n}(0), \dots, h_{k,n}(L-1)]^T$ , and where  $h_{k,n}(i) = \rho_{k,n}(i) \alpha_{k,n}(i)$ .

**Proof:** It is assumed that each transmit antenna acquires  $\bar{L}$  taps of CIR from each mobile device. Let  $i_{0,k,n}, \dots, i_{\bar{L}-1,k,n}$  be indices of selected taps. The peak power concentrated is

$$P_{peak}(\bar{L}) = \sum_{n=1}^{N_t} \sum_{s=0}^{\bar{L}-1} \alpha_{k,n}(i_{s,k,n})^2, \quad (16)$$

while the average power is

$$\bar{P}_{peak}(\bar{L}) = \sum_{n=1}^{N_t} \sum_{s=0}^{\bar{L}-1} E\{\alpha_{k,n}(i_{s,k,n})^2\} = N_t \sum_{s=0}^{\bar{L}-1} \Omega \gamma^{i_{s,k,n}}. \quad (17)$$

Since  $\gamma = e^{-\Delta/\Gamma} < 1$ , it is easy to show that  $i_{s,k,n} = s$  maximizes the average power concentration. Thus, the PPR scheme is optimal in maximizing the average peak power under the imposed feedback constraint. Also, it can be shown that the eigenvalues of  $\mathbf{R}_{\mathbf{h}_{k,n}}$  are  $\lambda_0 = \Omega, \dots, \lambda_{L-1} = \Omega \gamma^{L-1}$ . Thus,  $\bar{P}_{peak}^{(p)}(\bar{L}) = N_t \sum_{i=0}^{\bar{L}-1} \lambda_i$ .  $\square$

When all taps are available at the transmitter, the average power at the peak is equal to

$$\bar{P}_{peak}^{(p)}(L) = N_t \Omega \frac{1 - \gamma^L}{1 - \gamma} \Big|_{\bar{L}=L} \approx N_t \frac{\Omega}{1 - \gamma}. \quad (18)$$

Thus, we can characterize the performance degradation due to incomplete tap feedback as the ratio of (15) and (18), *i.e.*,

$$\mu(\bar{L}) = \frac{\bar{P}_{peak}^{(p)}(\bar{L})}{\bar{P}_{peak}^{(p)}(L)} = 1 - \gamma^{\bar{L}}. \quad (19)$$

As shown in Eq. (15), there is a spatial diversity gain,  $N_t$ , at the peak of the resultant channel. It can also be shown that the average power of the sidelobe remains the same no matter what  $N_t$  is. Thus, when more transmit antennas are used, the channel delay spread and ISI are reduced for a fixed symbol interval and the data rate is enhanced with respect to a fixed noise margin.

#### B. Phase-assisted Tap Selection

In the proposed system, the channel is assumed to be static for a relatively long period of time. Mathematically, parameters  $\alpha_{k,n}(i)$  and  $\rho_{k,n}(i)$  in the channel model remain unchanged. Rather than delivering the amplitude and the phase information together, we can send the amplitude and the phase data back to the transmitter separately. In this subsection, we will argue that, if the phase information is available at both sides, the transmission of the amplitude information can be made more easily with the phase as the side information. Since it takes only one bit to represent each phase, transmission of all phase information to the transmitter is cost effective.

Let  $\mathbf{h}_{k,n} = [h_{k,n}(0), \dots, h_{k,n}(L-1)]^T = \mathbf{P}_{k,n} \mathbf{a}_{k,n}$  where  $\mathbf{P}_{k,n} = \text{diag}[\rho_{k,n}(0), \dots, \rho_{k,n}(L-1)]$  and  $\mathbf{a}_{k,n} = [\alpha_{k,n}(0), \dots, \alpha_{k,n}(L-1)]^T$ . We will form an channel approximation as follows. Let

$$\hat{\mathbf{h}}_{k,n} = \mathbf{P}_{k,n} \hat{\mathbf{a}}_{k,n}. \quad (20)$$

This implies that the phase information is *completely* kept in the approximated channel. In other word, there is no loss in the phase. The approximation of amplitude  $\mathbf{a}_{k,n}$  is accomplished by applying the well-known K-L expansion. Let the eigen-decomposition of autocorrelation matrix  $\mathbf{R}_{\mathbf{a}_{k,n}} = E\{\mathbf{a}_{k,n} \mathbf{a}_{k,n}^H\} = \mathbf{E} \mathbf{\Lambda} \mathbf{E}^H$ , where  $\mathbf{\Lambda} = \text{diag}[\bar{\lambda}_0, \dots, \bar{\lambda}_{L-1}]$ ,  $\bar{\lambda}_0 \geq \dots \geq \bar{\lambda}_{L-1}$ , and superscript  $H$  is the Hermitian transpose operation. Then,  $\hat{\mathbf{a}}_{k,n}$  can be approximated by

$$\hat{\mathbf{a}}_{k,n} = \mathbf{E} \hat{\mathbf{w}}_{k,n}, \quad (21)$$

where  $\hat{\mathbf{w}}_{k,n} = [(\mathbf{E}(:, 1 : \bar{L})^H \mathbf{a}_{k,n})^T, 0, \dots, 0]^T$  and  $\mathbf{E}(:, 1 : \bar{L})$  is the matrix formed by the first  $\bar{L}$  columns of  $\mathbf{E}$ . It can be shown that the correlation matrix of  $\mathbf{a}_{k,n}$  has the following form

$$\mathbf{R}_{\mathbf{a}_{k,n}} = \Omega \begin{bmatrix} 1 & ab & \dots & ab^{L-1} \\ ab & b^2 & \dots & ab^L \\ \vdots & \vdots & \ddots & \vdots \\ ab^{L-1} & ab^L & \dots & b^{2(L-1)} \end{bmatrix}, \quad (22)$$

where  $a = \pi/4$  and  $b = \gamma^{1/2}$ . Since the above matrix is completely determined by  $\Omega$ ,  $\Delta$  and  $\Gamma$ , both sides can individually generate a set of KL bases without calculating  $\mathbf{R}_{\mathbf{a}_{k,n}}$  explicitly. Furthermore,  $\Delta$  is often fixed by the design specification, we may pre-calculate and store eigenbases in tables at the receiver for given  $\Delta$  and four different channel models (CM1-CM4) to save the computational power needed for the eigen-decomposition of  $\mathbf{R}_{\mathbf{a}_{k,n}}$ .

Since UWB has a good ranging capability [6], it is possible for the transmitter to figure out the distance and tell the receiver which bases to use. The receiver can pass the first  $\bar{L}$  KL coefficients corresponding to the  $\bar{L}$  largest bases, *i.e.*,  $\mathbf{E}(:, 1 : \bar{L})^H \mathbf{a}_{k,n}$  to the transmitter. The channel information can be synthesized at the other side via

$$\hat{\mathbf{h}}_{k,n} = \mathbf{P}_{k,n} \mathbf{E} \hat{\mathbf{w}}_{k,n} = \mathbf{P}_{k,n} \hat{\mathbf{a}}_{k,n}. \quad (23)$$

By following a similar procedure, we can show that the taps selected by PPR is the KL approximation of  $\mathbf{h}_{k,n}$ . From (23), we may view  $\hat{\mathbf{h}}_{k,n}$  as a linear combination of columns of  $\mathbf{P}_{k,n} \mathbf{E}$ .

#### IV. PERFORMANCE ANALYSIS OF PHASE-ASSISTED TAP SELECTION

The KL bases for the PPR scheme are deterministic. In contrast, matrix  $\mathbf{P}_{k,n} \mathbf{E}$  for the phase-assisted tap selection scheme has a set of random bases depending on the phase of each channel realization. The performance of the proposed scheme will be analyzed and shown to outperform the PPR scheme in this section. First, we prove the following proposition.

**Proposition 2.** The average power concentrated at the peak of the resultant channel of the proposed phase-assisted tap selection scheme is equal to

$$\bar{P}_{peak}^{(s)} = N_t \sum_{i=0}^{\bar{L}-1} \bar{\lambda}_i. \quad (24)$$

**Proof:** First, consider the case  $n = 1$  and the antenna index is discarded for notational simplicity. The average power at the peak of the resultant channel is

$$\begin{aligned} \bar{P}_{peak}^{(s)} &= E\left\{\left(\frac{\hat{\mathbf{h}}_k}{\|\hat{\mathbf{h}}_k\|}\right)^H \mathbf{h}_k\right\}^2 \\ &= E\left\{\left(\frac{\mathbf{P}_k \hat{\mathbf{a}}_k}{\|\mathbf{P}_k \hat{\mathbf{a}}_k\|}\right)^H (\mathbf{P}_k \mathbf{a}_k)\right\} = E\left\{\left(\frac{\hat{\mathbf{a}}_k}{\|\hat{\mathbf{a}}_k\|}\right)^T \mathbf{a}_k\right\}^2 \\ &= E\left\{\frac{\hat{\mathbf{a}}_k^H \mathbf{E} \mathbf{E}^H \mathbf{a}_k}{\hat{\mathbf{a}}_k^H \mathbf{E} \mathbf{E}^H \hat{\mathbf{a}}_k}\right\} = E\left\{\frac{(\hat{\mathbf{w}}_k^T \mathbf{w}_k)^2}{\hat{\mathbf{w}}_k^T \hat{\mathbf{w}}_k}\right\} \\ &= \text{tr} E\left\{\mathbf{E}(:, 1 : \bar{L})^H \mathbf{a}_k \mathbf{a}_k^T \mathbf{E}(:, 1 : \bar{L})\right\} \\ &= \text{tr}\left\{\mathbf{E}(:, 1 : \bar{L})^H \mathbf{R}_{\mathbf{a}_k} \mathbf{E}(:, 1 : \bar{L})\right\} = \sum_{i=0}^{\bar{L}-1} \bar{\lambda}_i, \end{aligned}$$

where  $\text{tr}\{\mathbf{A}\}$  is the trace of matrix  $\mathbf{A}$ . Using the same approach, we can show that, when there are  $N_t$  antennas, the average power concentrated at the peak is  $N_t \sum_{i=0}^{\bar{L}-1} \bar{\lambda}_i$ .  $\square$

**Proposition 3.** Let  $\lambda_i$  and  $\bar{\lambda}_i$  denote the  $i$ th eigenvalues of  $\mathbf{R}_{\mathbf{h}_{k,n}}$  and  $\mathbf{R}_{\mathbf{a}_{k,n}}$ , respectively. We have  $\lambda_i > \bar{\lambda}_i$ ,  $1 \leq i \leq L-1$ , and  $\lambda_0 \geq \lambda_0$ .

**Proof:** Note that  $\mathbf{R}_{\mathbf{a}_{k,n}}$  can be decomposed as

$$\mathbf{R}_{\mathbf{a}_{k,n}} = \Omega(1-a) \text{diag}[1, b^2, \dots, b^{2(L-1)}] + \Omega a \mathbf{b} \mathbf{b}^T, \quad (25)$$

where  $\mathbf{b} = [1, b, \dots, b^{L-1}]^T$  and  $a$  and  $b$  are defined earlier. According to Theorem 8.1.8 in [7], the eigenvalue of  $\mathbf{R}_{\mathbf{a}_{k,n}}$  is bounded by

$$\hat{\lambda}_i \leq \bar{\lambda}_i \leq \hat{\lambda}_{i-1} \quad i = 1, \dots, L-1, \quad (26)$$

where  $\hat{\lambda}_i = \Omega(1-a)b^{2i}$ . First, we assume that  $\bar{\lambda}_i \geq \lambda_i$  is true for any  $1 \leq i \leq L-1$ . Then, we have

$$\Omega(1-a)b^{2(i-1)} \geq \bar{\lambda}_i \geq \lambda_i = \Omega b^{2i}. \quad (27)$$

Substituting  $a = \pi/4$  and  $b = \gamma^{1/2} = e^{-\Delta/2\Gamma}$  into (27), we get

$$\frac{\Delta}{\Gamma} \geq -\ln\left(1 - \frac{\pi}{4}\right) \approx 1.54. \quad (28)$$

In our current system setting, the multipath resolution must be less than the decay time constants given in [4] due to the narrow time-domain pulse width. Thus, the assumption made is not true, which implies  $\lambda_i > \bar{\lambda}_i$  for all  $1 \leq i \leq L-1$ . Finally, since

$$\sum_{i=0}^{L-1} \lambda_i = \text{tr}(\mathbf{R}_{\mathbf{h}_{k,n}}) = \text{tr}(\mathbf{R}_{\mathbf{a}_{k,n}}) = \sum_{i=0}^{L-1} \bar{\lambda}_i,$$

we conclude that  $\bar{\lambda}_0 \geq \lambda_0$ .  $\square$

**Proposition 4.** The phase-assisted channel feedback scheme concentrates more power than the PPR scheme if the same number of taps or coefficients is fed back from the transmitter to the receiver.

**Proof:** The concentrated power for these two methods are

$$\bar{P}_{peak}^{(p)} = N_t \sum_{i=0}^{\bar{L}-1} \lambda_i = N_t \left( \sum_{i=0}^{L-1} \lambda_i - \sum_{i=\bar{L}}^{L-1} \lambda_i \right), \quad (29)$$

and

$$\bar{P}_{peak}^{(s)} = N_t \sum_{i=0}^{\bar{L}-1} \bar{\lambda}_i = N_t \left( \sum_{i=0}^{L-1} \bar{\lambda}_i - \sum_{i=\bar{L}}^{L-1} \bar{\lambda}_i \right), \quad (30)$$

respectively. From **Proposition 3**, we have that  $\sum_{i=\bar{L}}^{L-1} \bar{\lambda}_i < \sum_{i=\bar{L}}^{L-1} \lambda_i$  for  $\bar{L} > 1$ . Since  $\sum_{i=0}^{L-1} \bar{\lambda}_i = \sum_{i=0}^{L-1} \lambda_i$ , we conclude that  $\bar{P}_{peak}^{(s)} > \bar{P}_{peak}^{(p)}$ .  $\square$

As compared with the PPR scheme that sends the limited phase and the amplitude information, the proposed phase-assisted tap selection method keeps the phase information unchanged and approximates the amplitude with partial KL coefficients to provide a more detailed picture about the channel. Additional computational complexity is required to calculate  $\mathbf{E}(:, 1 : \bar{L}) \mathbf{a}_{k,n}$  for every package. However, it is observed that

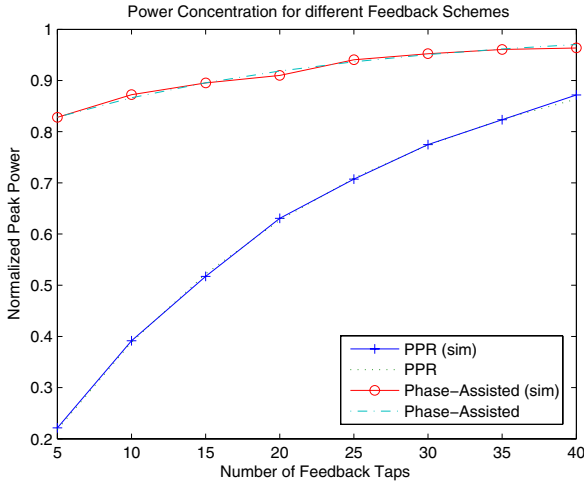


Fig. 1. The normalized peak power as a function of the selected tap number used in the feedback, where  $N_t=1$ ,  $\Delta=1\text{ns}$ ,  $\Gamma=20.5\text{ns}$  (CM3).

the peak power concentration for the case with  $N_t = 1$  and  $\bar{L} = 1$  is greater than 78% of the total channel power. In the traditional RAKE receiver, more than 20 correlation operators are needed to achieve such a high power concentration for every transmitted symbol. Therefore, the proposed method is more efficient than the PPR scheme and the RAKE receiver.

## V. SIMULATION RESULTS

Computer simulation was conducted to study the performance of different TRP-based DSUWB systems. In the first experiment, we examined the power concentration for the PPR scheme and the phase-assisted tap selection scheme. The concentrated power at the peak is normalized by the total channel power in Fig. 1, where the simulation result is obtained as an average over 1000 channel realizations and the theoretical result is also plotted for comparison. We see from this figure that the simulation and theoretical results match very well. Furthermore, the phase-assisted tap selection scheme outperforms the PPR scheme at all selected tap numbers. A larger gain is observed when the selected tap number is smaller.

Next, we study the bit error rate (BER) performance of both systems in Fig. 2, where  $N_u = 3$ ,  $\bar{L} = 10$ ,  $\Delta = 1\text{ns}$ ,  $\Gamma = 20.5\text{ns}$  (CM3),  $T_s = L\Delta\text{ns}$ . The channel remains unchanged for one data package which contains  $10^3$  symbols and then changes independently from package to package. The result shown here is the average over 100 channel realizations. Again, the phase-assisted tap selection method has better performance. The gain from multiple RAKE fingers is smaller when the phase information is available. This may be explained by that the sidelobe contributes less signal power when the power is more concentrated at the peak. It is also observed that multiple transmit antennas provides a spatial diversity gain that is larger as  $E_b/N_o$  goes up. This can be explained below. When the signal power is large, system performance is mainly determined by interference rather than background noise. The spatial diversity that enhances power concentration at the peak rather than sidelobe leads to smaller interference and therefore better performance.

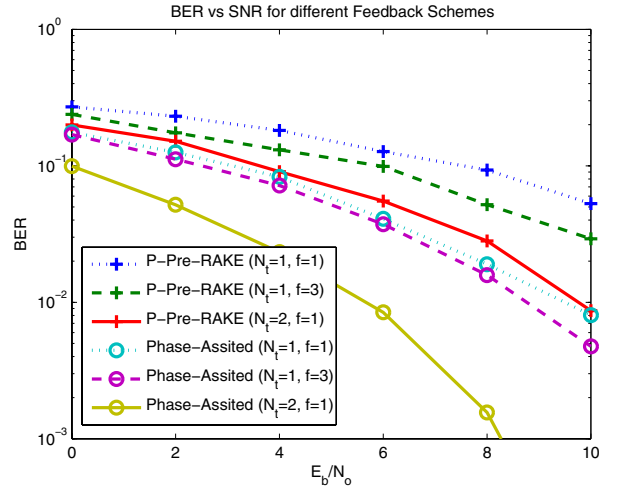


Fig. 2. Comparison of the bit error rate performance for two tap selection schemes with various transmit antennas and rake fingers.

## VI. CONCLUSION

The TRP-based DSUWB system concentrates the received signal power by shifting the hardware complexity from the receiver to the transmitter. The use of multiple transmit antennas provides better power focusing and leads to a simpler receiver design. The availability of the channel information is the key to this system. We analyzed the performance of a known PPR scheme and proposed a new method called the phase-assisted tap selection scheme. The proposed new scheme is effective in received power focusing. It outperforms the traditional PPR scheme, which was proved analytically and verified by computer simulation.

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