Throughput Analysis of Feedback-directed Adaptive MIMO-OFDM Systems

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***Abstract*— The throughput performance of adaptive multi-input multi-output orthogonal frequency division multiplexing (MIMO-OFDM) system without channel station information (CSI) in priori is investigated in this research. First, throughput scaling factor (pre-log factor) of an n_t transmit antennas, n_r receive antennas MIMO-OFDM with total N subcarriers is asymptotically equal to $n^*(1-n^*\frac{L}{MN})$, where $n^* = \min(n_t, n_r)$ and M is the number of OFDM blocks experienced a constant channel. Second, feedback directed adaptive schemes can achieve the pre-log factor of the lower bound. Third, diversity-multiplexing gain tradeoff of variable-rate systems has been shown to have the same behavior as MIMO systems employed with random Gaussian codewords. Finally, it is shown that throughput of OFDMA with opportunistic scheduling performs better than traditional OFDM-TDMA scheduling with constant enhancement independent of SNR in multiuser downlink environment.**

I. Introduction

Adaptive modulation using variable-rate-variable-power (VRVP) QAM was originally proposed and studied in [1]. A general framework of MIMO adaptive modulation was presented and analyzed in [2] based on transmitter beamforming and VRVP QAM. Alternative solution to this problem is the "mercury/water-filling" strategy based on the universal formula of mutual information and minimum-mean-square-errors (MMSE) [3], where the "mercury" is to compensate the gap between discrete constellations and the Gaussian input. Adaptive bit-interleaved coded modulation was proposed in [4] to provide a reliable adaptive transmission scheme when outdated channel information is used. More recently, bit-interleaved coded multiple beamforming (BICMB) [5] has been proposed and shown to achieve the full spatial multiplexing gain while maintaining full diversity. Pioneering work on applying adaptive modulation to MIMO-OFDM systems can be found in [6], [7] and the references therein.

In this work, we consider an $n_t \times n_r$ MIMO-OFDM system with N subcarriers. Following the techniques developed in [8], we first derive theoretical capacity provided by an MIMO-OFDM system without channel state information (CSI) at the receiver end. We show that, in the high SNR region, the lower bound of noncoherent capacity is asymptotically equal to $n^*(1-n^*\frac{L}{N})\log\rho$, where n^* is $\min(n_t, n_r)$, L is the number of channel taps, and ρ is the signal-to-noise ratio (SNR). A similar result was also reported in [9], where the behavior of noncoherent capacity of a wideband MIMO-OFDM was analyzed as a function of the total bandwidth. Then, we propose an adaptive bit-interleaved coded modulation scheme to achieve the noncoherent capacity while still maintain consistently reliable performance. Performance comparison of two multiple access schemes of OFDM namely, OFDM-TDMA and OFDMA in multiuser downlink environment has been reported in recent work [10] [11]. We generalize the analysis to multiple antennas and derives theoretical throughput of these two schemes. It is shown that OFDMA with opportunistic scheduling performs better than traditional OFDM-TDMA scheduling with a constant enhancement dependent on number of antennas and users but independent of SNR.

The rest of this chapter is organized as follow. A general MIMO-OFDM system over time-varying frequency-selective fading channels is discussed in Sec. II. The transceiver of adaptive MIMO-OFDM system using transmitter beamforming is presented in Sec. III, where three adaptive squared QAM schemes are considered. Theoretically achievable throughput provided by adaptive MIMO-OFDM system without the knowledge of channel state information (CSI) is analyzed in Sec. IV, where noncoherent capacity is first derived , followed by the throughput analysis of adaptive schemes. The behavior of diversity-multiplexing gain tradeoff is also analyzed. Throughput comparison of OFDMA employed with opportunistic scheduling with OFDM-TDMA scheduling in MIMO multiuser downlink environment is analyzed in Sec. V. Concluding remarks are given in Sec. VII.

II. System Model

We consider a MIMO-OFDM system with n_t transmit antennas and n_r receive antennas. Let N be the total number of subcarriers and $\mathbf{s}_i = [s_{i,0}, \ldots, s_{i,N-1}]^T$ the frequency-domain symbols sent by the ith antenna. The corresponding time-domain vector is given by $\mathbf{x}_i = \mathbf{W}^H\mathbf{s}_i$, where $\mathbf{W}$ is the N-point discrete Fourier transform (DFT) matrix. Let the discrete-time channel response from the ith antenna to the jth antenna be $\mathbf{h}_{i,j} = [h_{i,j,0}, \cdots, h_{i,j,L-1}]^T$ where L is the maximum channel length. The covariance matrix of channel taps is $\Sigma_{\mathbf{h}_{ij}} = \mathrm{E}(\mathbf{h}_{i,j}\mathbf{h}_{i,j}^H)$. The Rayleigh fading is assumed and $\{\mathbf{h}_{i,j}\}_{i=1,j=1}^{n_t,n_r}$ are i.i.d zero-mean complex Gaussian random vectors with covariance matrix $\Sigma_{\mathbf{h}_{ij}} = \Sigma_{\mathbf{h}}$. Under the wide-sense-stationary-uncorrelated-scattering (WSSUS) assumption, $\Sigma_{\mathbf{h}}$ is a diagonal matrix with $[\sigma_0^2, \sigma_1^2, \cdots, \sigma_{L-1}^2]^T$ as diagonal entries with $\sum_{l=0}^{L-1}\sigma_l^2 = \sigma_h^2$. A cyclic prefix (CP) of length $L_{CP} > L$ is appended in front of $\mathbf{x}_i(m)$ to eliminate the interblock interference (IBI). Assuming $N \gg L$, the loss of spectral efficiency due to CP inserted is neglected throughout the paper. The resulting vector $\mathbf{u}_i(m)$ with length $L_u = N + L_{CP}$ is then transmitted over the channel.

After matched filtering and sampling and CP removal, the frequency domain signal, which is obtained by applying DFT to the received discrete time vector, can be represented as

$$\mathbf{y}^{(k)} = \mathbf{H}^{(k)}\mathbf{s}^{(k)} + \mathbf{n}^{(k)}, \tag{1}$$

where $\mathbf{y}^{(k)}$ and $\mathbf{s}^{(k)}$ are the $n_r \times 1$ output vector and $n_t \times 1$ input vector, respectively, $\mathbf{H}^{(k)}$ is the $n_r \times n_t$ channel frequency response matrix corresponding to the kth subcarrier and the mth OFDM symbol and it is easy to verified that the entries of $\mathbf{H}^{(k)}$ are $\mathcal{CN}(\mathbf{0}, \sigma_h^2\mathbf{I})$ distributed. Let AWGN noise vectors $\{\mathbf{n}^{(k)}\}$ be i.i.d. $\mathcal{CN}(\mathbf{0}, \sigma^2\mathbf{I})$ distributed.

III. Feedback-directed Adaptive MIMO-OFDM System

Let the singular value decomposition of $\mathbf{H}^{(k)}$ be $\mathbf{H}^{(k)} = \mathbf{U}^{(k)}\mathbf{D}^{(k)}\mathbf{V}^{(k)H}$, where $\mathbf{U}^{(k)}$ and $\mathbf{V}^{(k)}$ are unitary matrices, and $\mathbf{D}^{(k)}$ is a diagonal matrix with diagonal entries $\{d_i^{(k)}\}_{i=1}^{n^*}$. Multiplying (1) by $\mathbf{U}^{(k)H}$, we get

$$\mathbf{U}^{(k)H}\mathbf{y}^{(k)} = \mathbf{y}^{(k)\prime} = \mathbf{D}^{(k)}\mathbf{s}^{(k)\prime} + \mathbf{n}^{(k)\prime}, \tag{2}$$

where $\mathbf{s}^{(k)\prime} = \mathbf{V}^{(k)H}\mathbf{s}^{(k)}$ and $\mathbf{n}^{(k)\prime} = \mathbf{U}^{(k)H}\mathbf{n}^{(k)}$. Please note that $\mathbf{s}^{(k)\prime}$ and $\mathbf{n}^{(k)\prime}$ have the same distribution as $\mathbf{s}^{(k)}$ and $\mathbf{n}^{(k)}$, respectively, due to the unitary property of $\mathbf{U}^{(k)}$ and $\mathbf{V}^{(k)H}$. Thus, we have decomposed the MIMO channel for a particular tone into n^* parallel subchannels, and there are totally n^*N subchannels in a MIMO-OFDM systems.

The bit-error-rate (BER) of uncoded square QAM constellation of size 2^r, where r is an even integer, can be approximated by [1]

$$\mathrm{BER_{QAM}} \approx 0.2\exp\left(\frac{-1.5\gamma}{2^r - 1}\right), \tag{3}$$

where γ is the received SNR per symbol. Given a target BER at $\mathrm{BER_t}$, the number of bits per constellation symbol can be determined (3). Let the power and the constellation rate allocated at the i-th subchannel of the k-th subcarrier be $P_i^{(k)}$ and $r_i^{(k)}$, respectively. The optimization problem can be represented as $\max_{r_i^{(k)}} \sum_{i,k} r_i^{(k)}$, subject to $\sum_{i,k} P_i^{(k)} \leq P, \mathrm{BER}_i^{(k)} \leq \mathrm{BER_t}, r_i^{(k)} \geq 0,\ P_i^{(k)} \geq 0$. Several algorithms have been proposed to solve this variable-rate-variable-power (VRVP) problem; for example, methods described in [1], [2]. However, the complexity are too high to be adopted in a real-time adaptive system. Two suboptimum but feasible solutions are variable-rate (VR) scheme and variable-power (VP) scheme. In the VR QAM scheme, each subchannel is allocated with the same power; namely, $P_i^{(k)} = \frac{P}{n_t N}$, but with a variable constellation; on the other hand, a fixed constellation for all subcarriers in the VP QAM scheme. The selection of VR constellations based on $d_i^{(k)}$, BER_t and transmitter SNR ($P/n_t N\sigma^2$) is summarized in Table I. On the other hand, Table II presents the pseudo code of the VP adaptive scheme. It is easy to see that VRVP has the highest computational complexity, next VP and finally, VR. Note that, in the high SNR region, the capacity limit with CSI at the transmitter (waterfilling) is equal to that without CSI at the transmitter. Thus, VR has the same asymptotic performance as VRVP yet with a much lower complexity.

TABLE I
Adaptive VR QAM selection.

Size	d^2 Region, $K = \frac{\ln(0.2/\mathrm{BER_t})}{1.5}$
0	$d^2 < \frac{(Q^{-1}(\mathrm{BER_t}))^2}{P/n_t N\sigma^2}$
BPSK	$\frac{(Q^{-1}(\mathrm{BER_t}))^2}{P/n_t N\sigma^2} \leq d^2 < \frac{(Q^{-1}(\mathrm{BER_t}))^2}{P/2n_t N\sigma^2}$
4-QAM	$\frac{(Q^{-1}(\mathrm{BER_t}))^2}{P/2n_t N\sigma^2} \leq d^2 < \frac{15K}{P/n_t N\sigma^2}$
16-QAM	$\frac{15K}{P/n_t N\sigma^2} \leq d^2 < \frac{63K}{P/n_t N\sigma^2}$
64-QAM	$\frac{63K}{P/n_t N\sigma^2} \leq d^2 < \frac{255K}{P/n_t N\sigma^2}$
256-QAM	$\frac{255K}{P/n_t N\sigma^2} \leq d^2$

TABLE II
The pseudo code for the VP adaptive scheme.

1. Order $\{d_i^{(k)}\}$ from the largest to the smallest
The ordered set is written as $\{\tilde{d}_j\}_{j=1}^{n_t N}$,
where $(i,k) \leftrightarrow j$ is a 1 to 1 mapping.
2. For r=1,2,4,6,8
$j = 0$; $\tilde{P} = 0$; $\{\tilde{P}_{j,r}\}_{j=1}^{n_t N} = 0$;
if $\tilde{P} < P$
{ $j = j + 1$;
calculating $\tilde{P}_{j,r}$;
$\tilde{P} = \tilde{P} + \tilde{P}_{j,r}$};}
$\tilde{R}_r = (j-1)r$; $\tilde{P}_{j,r} = 0$;
3. Return $r^* = \arg\max_r \tilde{R}_r$ and set $\{P_i^{(k)}\} \leftrightarrow \{\tilde{P}_{j,r^*}\}_{j=1}^{n_t N}$.

However, there is no information of CSI at both transmitter and receiver end. The identification of all $n_t \times n_r \times L$ channel taps requires at least the same number of pilots, which implies that $n_r L$ pilots at each transmit antenna.. This overhead will cause a penalty on spectrum efficiency with ratio

$$\delta = \frac{n_r L}{MN} \tag{4}$$

It is well known that Cramer-Rao bound (CRB) decays with SNR, which implies that we can obtain reliable channel estimation in the high SNR region using the MMSE estimator.

IV. Throughput Limitation

A. Noncoherent Capacity and Throughput Analysis

If the CSI is unknown at the receiver, the noncoherent capacity can be represented as

$$C_{\mathrm{non}} = \sup_{p(\mathbf{S})} I(\mathbf{Y}, \mathbf{S}). \tag{5}$$

Using the chain rule, we can decompose the mutual information as

$$I(\mathbf{Y}, \mathbf{S}) = I(\mathbf{Y}, \mathbf{S}, \mathbf{H}) - I(\mathbf{Y}, \mathbf{H}|\mathbf{S}). \tag{6}$$

According to the property that mutual information is greater than conditional mutual information, we have $I(\mathbf{Y}, \mathbf{S}, \mathbf{H}) >$

$I(\mathbf{Y},\mathbf{S}|\mathbf{H})$. It is straightforward to show that

$$I(\mathbf{Y},\mathbf{H}|\mathbf{S}) \leq \frac{n_t n_r}{MN}\sum_{l=0}^{L-1}\log(1+\rho N\sigma_l^2). \qquad (7)$$

Substituting (7) into (6), we have the lower bound of the noncoherent capacity as

$$C_{\text{non}} \geq C_{\text{nonLB}} = C_{\text{coh}} - \frac{n_t n_r}{MN}\sum_{l=0}^{L-1}\log(1+\rho N\sigma_l^2). \qquad (8)$$

In the high SNR region, an improved lower bound can be obtained by turning off the "dispensable antennas", *i.e.*, only $n^* \times n^*$ channels being used. Then, we obtain that in the high SNR region, the lower bound is asymptotically equal to

$$\lim_{\rho\to\infty}\frac{C_{\text{nonLB}}}{\log\rho} = n^*(1-n^*\frac{L}{MN}), \qquad (9)$$

Since (9) is a second order polynomial, it can be maximized by choosing $n^* = \frac{MN}{2L}$ with the maximum lower bound equal to $\frac{MN}{4L}$. If $\frac{MN}{2L}$ is not an integer, we can choose the nearest integer to $\frac{MN}{2L}$ to be n^* to maximize the lower bound. Based on the above analysis, we have the following design rules for MIMO-OFDM systems.

R1 The number of transmit antennas, n_t, should not be larger than that of receive antennas, n_r.

R2 The maximum number of transmit antennas should not be larger than $MN/2L$.

Design rule **R1** suggests that we should only consider the case $n_t = n_r = n^*$ in the remaining part of this paper. Comparing (4) with (9), we conclude that adaptive schemes based on training can achieve the lower bound of noncoherent capacity if the pre-log factor is equal to n^* in data mode, which will be shown in the remaining of the section. According to the Proposition 1 in [12], the distribution of the $\mathbf{H}^{(k)^H}\mathbf{H}^{(k)}$ of any tone is identical independent distributed. Thus to find the average throughput, we can find the statistic of any particular tone. For notation simplification, we drop the superscript $^{(k)}$. We choose VR scheme as an example of analysis of the throughput of adaptive modulation. Let $\lambda_i = d_i^2$. Then λ_i are distributed according to the property of Wishart distribution because λ_i is eigenvalue of $\mathbf{H}^H\mathbf{H}$. The marginal pdf of any unordered eigenvalue has a closed form expression [13]

$$\begin{aligned} p_\lambda(\lambda) \quad &= \frac{1}{n^*\sigma_h^2}\times\sum_{i=0}^{n^*-1}\sum_{j_1=0}^{i}\sum_{j_2=0}^{i}(-1)^{j_1+j_2}\{ \\ &\frac{C_{j_1}^i C_{j_2}^i}{j_1!j_2!}(\frac{\lambda}{\sigma_h^2})^{j_1+j_2}e^{-\frac{\lambda}{\sigma_h^2}}\}, \end{aligned} \qquad (10)$$

where $C_j^i = i!/[(i-j)!j!]$. In an ideal continuous rate adaption scheme, the instantaneous rate can be obtained by substituting $\gamma = \frac{\lambda_i P}{n_t N\sigma^2}$ and $K = \frac{\ln(0.2/\text{BER}_\text{t})}{1.5}$ into (3),

$$r(\lambda_i) = \log(1+\frac{\lambda P}{Kn_tN\sigma^2}) = \log(1+\frac{\lambda_i\rho}{K}). \qquad (11)$$

The throughput per channel use of continuous rate adaption scheme can be obtained by averaging (11) with respect to the pdf (10):

$$\begin{aligned} R_{\text{cont.VR}} &= n^*\int_0^\infty \log(1+\frac{\lambda\rho}{K})p_\lambda(\lambda)d\lambda \qquad (12) \\ &\approx n^*(\log\rho + \mathrm{E}(\log\lambda) - \log K), \qquad (13) \end{aligned}$$

where (13) is a high SNR approximation. A constant penalty of $n^*\log K$ is appeared due to the target BER constraint. In practical, the rate is usually selected from square QAM constellations and the rate allocated to each subchannel can be found as

$$\begin{aligned} r(\lambda) = 2j, \qquad &\text{if } 2j \leq \log(1+\frac{\lambda P}{Kn_tN\sigma^2}) < 2(j+1), \\ \text{or equivalently} \qquad &\lambda_{L,j}(\rho) \leq \lambda < \lambda_{U,j}(\rho), \end{aligned} \qquad (14)$$

where $\lambda_{L,j}(\rho) = \frac{2^{2j}-1}{\rho/K}$ and $\lambda_{U,j}(\rho) = \frac{2^{2(j+1)}-1}{\rho/K}$. The throughput per channel use of discrete rate adaption scheme with J constellation size of square QAM can be obtained similarly by averaging (14) with respect to the pdf (10):

$$\begin{aligned} R_{\text{dis.VR}} &= n^*\sum_{j=1}^{J}\int_{\lambda_{L,j}(\rho)}^{\lambda_{U,j}(\rho)} 2jp_\lambda(\lambda)d\lambda \\ &+n^*\int_{\lambda_{U,J}(\rho)}^{\infty} 2Jp_\lambda(\lambda)d\lambda, \end{aligned}$$

which is strictly upper-bounded by (12).

For example, we set $M=1$, $\sigma_h^2=1$, $N=64$, $L=4$ and $n_t=n_r=n^*=2$ with a target BER at 10^{-3}. The receiver does not have the knowledge of CSI so that we reserve $n_r \times L = 8$ pilots from a total of $N=64$ subcarriers for the training and tracking purpose. Fig. 1 depicts the behavior of coherent capacity, C_{coh}, lower bound of noncoherent capacity, C_{nonLB}, throughput of constant VR scheme, $R_{\text{cont.VR}}$, throughput of discrete VR scheme with largest constellation size as 1024-QAM, $R_{1024\text{VR}}$, and throughput of discrete VR scheme with largest constellation size as 256-QAM, $R_{256\text{VR}}$ as the function of transmit SNR ρ. It is observed that the gap between the lower bound of nocoherent capacity and coherent capacity increases as SNR becomes larger. This is due to a penalty of L/N paid as shown in (9), which reduces the slope from $n_t = 2$ to $n_t(1-n_r\frac{L}{N}) = 1.75$. We see that all VR schemes can not achieve noncoherent capacity at any SNR due to the constant term $n^*\log K = 3.64$ from BER constraint (13). It implies that to further increase VR throughput, power allocation is required to compensate this BER constrained penalty. Comparing curve $R_{1024\text{VR}}$ with $R_{256\text{VR}}$, we observe that the throughput are almost the same when SNR≤25dB; however, the gap between them becomes larger when SNR goes even larger. It is because there is good chance for subcarrier to support higher rate in high SNR region.

B. Diversity-Multiplexing Tradeoff

Let R be the rate of information transmission in bits per channel use. Let the spatial multiplexing gain m be defined

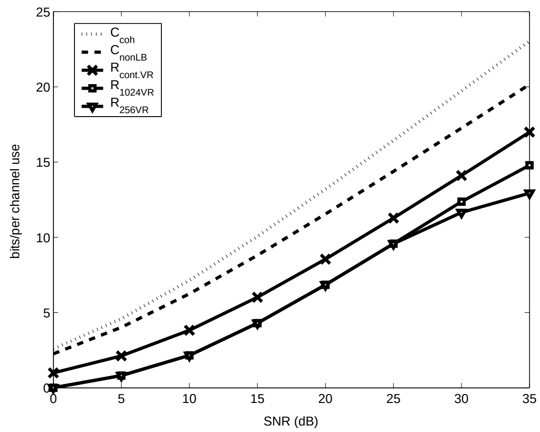

Fig. 1. Throughput of VR schemes of MIMO-OFDM systems.

by:

$$R = m \log \rho,$$

where ρ is the transmitted SNR. The outage corresponds to the event

$$\log(\prod_{i=1}^{n_t}(1+\frac{\lambda_i \rho}{K})) < m \log \rho. \quad (15)$$

The joint pdf of ordered $\{\lambda_i\}$ ($\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_{n_t}$) for a $n_t \times n_r$ ($n_r \leq n_t$) MIMO systems is

$$p_{\underline{\lambda}}(\underline{\lambda}) = c_1 \exp(-\frac{1}{\sigma_h^2}\sum_{i=1}^{n_t}\lambda_i)\prod_{i=1}^{n_t}\lambda_i^{n_r-n_t}\prod_{i<j}(\lambda_i-\lambda_j)^2,$$

where c_1 is a normalized constant dependent on n_r, n_r and σ_h^2. Define

$$\lambda_i = K\rho^{-\alpha_i}.$$

Following the techniques in [14], the condition of outage (15) becomes

$$\sum_{i=1}^{n_t}\min(\alpha_i, 1) > n_t - m. \quad (16)$$

Our goal is to derive the expression for the probability $P_{out}(m)$ of outage when one is attempting to communicate at rate $R = m \log \rho$ bits per channel use of the form:

$$P_{out}(m) \doteq_{\rho\to\infty} \rho^{-d_{out}(m)},$$

where

$$d_{out}(m) = \lim_{\rho\to\infty}\frac{-\log(P_{out}(m))}{\log \rho}$$

is referred to the diversity gain [14]. According to (16), the outage probability is equal to

$$P_{out}(m) = \Pr\{\sum_{i=1}^{n_t}\min(\alpha_i,1) > n_t - m\}.$$

It is straightforward to show that $p_{\underline{\alpha}}(\underline{\alpha})$ is equal to

$$c_2 \exp(-\frac{K}{\sigma_h^2}\sum_{i=1}^{n_t}\rho^{-\alpha_i})\prod_{i=1}^{n_t}\rho^{-\alpha_i(n_r-n_t+2i-1)}(\log\rho)^{n_t}, \quad (17)$$

where c_2 is a normalized constant dependent on K, n_t, n_r and σ_h^2. Then the outage probability is

$$P_{out}(m) = \int\cdots\int_{\sum\min(\alpha_i,1)>n_t-m} p(\underline{\alpha})d\underline{\alpha}.$$

The following observations facilitate the determination of diversity gain $d_{out}(m)$.

1) The contribution from $(\log\rho)^{n_t}$ and c_2 in (17) to $P_{out}(m)$ can be ignored after taking the logarithm and comparing it with $\log\rho$.
2) The presence of $\exp(-\frac{K}{\sigma_h^2}\sum_{i=1}^{n_t}\rho^{-\alpha_i})$ in (17) implies $\alpha_i \geq 0$.
3) $d_{out}(m)$ should be the dominant exponent in (17).

Thus, we conclude that the diversity gain is equal to

$$d_{out}(m) = \inf_{\alpha_i\geq 0, \sum\min(\alpha_i,1)>n-m}\sum_{i=1}^{n_t}\alpha_i(2i-1+n_r-n_t).$$

For m be an integer, the infimum corresponding to choosing $\alpha_i = 1, i \leq n_t - r$ and $\alpha_i = 0, n_t - m < i < n_t$, and

$$d_{out}(m) = (n_t - m)(n_r - m), \quad (18)$$

which has the same behavior as the MIMO systems employed with Gaussian random codeword [14] despite of the constant penalty resulted from BER constraint. Taking the training into account, the condition of outage will become

$$\sum_{i=1}^{n_t}\min(\alpha_i,1) > n - \frac{m}{1-\delta},$$

where δ is defined in (4). The diversity gain in this system is

$$d_{out}(m) = \inf_{\alpha_i\geq 0, \sum\min(\alpha_i,1)>n-\frac{m}{1-\delta}}\sum_{i=1}^{n_t}\alpha_i(2i-1+n_r-n_t).$$

The infimum corresponds to choosing

$$\alpha_i = \begin{cases} 1, & 1 \leq i \leq n_t - \lceil\frac{m}{1-\delta}\rceil; \\ \lceil\frac{r}{1-\delta}\rceil - \frac{m}{1-\delta}, & i = n_t - \lfloor\alpha_i\rfloor; \\ 0, & n_t - \lfloor\alpha_i\rfloor < i \leq n_t. \end{cases} \quad (19)$$

The diversity order is obtained by straight-line interpolation between two points: $(\lfloor\frac{m}{1-\delta}\rfloor, (n_t - \lfloor\frac{m}{1-\delta}\rfloor)(n_r - \lfloor\frac{m}{1-\delta}\rfloor))$ and $(\lceil\frac{m}{1-\delta}\rceil, (n_t - \lceil\frac{m}{1-\delta}\rceil)(n_r - \lceil\frac{m}{1-\delta}\rceil))$. Figure 2 depicts the behavior of diversity-multiplexing gain tradeoff of VR scheme with channel known (dashed line) or unknown (solid line) in priori.

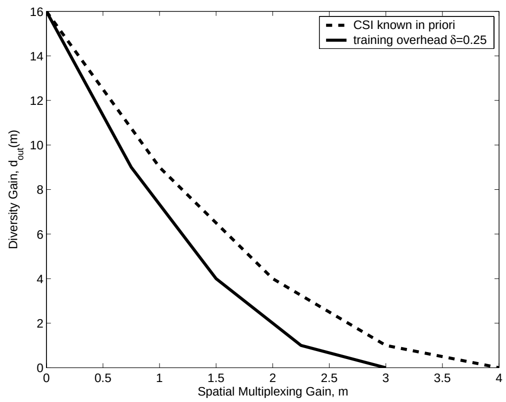

Fig. 2. Diversity-multiplexing gain tradeoff of VR scheme.

V. Throughput for Multiuser Downlink

First, we consider a 2×2 MIMO-OFDM with U users whose channel taps are i.i.d distributed with $\sigma_h^2 = 1$.. The joint pdf of any unordered eigenvalue (λ_1, λ_2) of this special case can be obtained from (10)

$$p_{\lambda_1,\lambda_2}(\lambda_1, \lambda_2) \quad = \quad (2K_{2,2})^{-1} e^{-(\lambda_1+\lambda_2)}(\lambda_1 - \lambda_2)^2. \tag{20}$$

Let $\eta = \lambda_1\lambda_2$. According to (11),

$$\begin{aligned} r_1 + r_2 \quad &= \quad \log(1 + \frac{\lambda_1 P}{2KN\sigma^2}) + \log(1 + \frac{\lambda_2 P}{2KN\sigma^2}) \\ &\approx \quad \log(\lambda_1\lambda_2 \times (\frac{P}{2KN\sigma^2})^2) \end{aligned}$$

Thus, the strongest channel gain defined in continuous VR scheme is equal to the largest η among the users. The pdf of η is

$$\begin{aligned} p_\eta(\eta) \quad &= \quad \int_0^\infty \frac{1}{\lambda_2} p_{\lambda_1,\lambda_2}(\frac{\eta}{\lambda_2}, \lambda_2) d\lambda_2 \\ &= \quad \int_0^\infty (2K_{2,2}\lambda_2)^{-1} e^{-(\frac{\eta}{\lambda_2}+\lambda_2)} (\frac{\eta}{\lambda_2} - \lambda_2)^2 d\lambda_2 \end{aligned} \tag{21}$$

The pdf of the largest η, η^*, is then equal to,

$$p_{\eta^*}(\eta^*) = U p_\eta(\eta^*)(1 - \int_0^{\eta^*} p_\eta(\eta) d\eta)^{U-1}. \tag{22}$$

The throughput of the 2×2 MIMO OFDM with multiuser diversity is given by

$$\begin{aligned} R_{\text{cont.VR,OPP},2\times2} \quad &\approx \quad \int_0^\infty \log(\eta^* \times (\frac{P}{2KN\sigma^2})^2) p_{\eta^*}(\eta^*) d\eta^* \\ &= \quad 2(\log\rho - \log K) + \mathrm{E}(\log\eta^*). \end{aligned} \tag{23}$$

In general, to find downlink throughput of an $n \times n$ MIMO-OFDM system with U users employed with continuous VR scheme, we first define $\eta_i = \prod_{j=1}^n \lambda_{i,j}$, where $\lambda_{i,j}$ is the j-th eigenvalue of $i-$th user. Let $\eta^* = \max\{\eta_i\}_{i=1}^U$, then the throughput of multiuser OFDM downlink with opportunist scheduling is given by

$$R_{\text{cont.VR,OPP}} \approx n(\log\rho - \log K) + \mathrm{E}(\log\eta^*). \tag{24}$$

Comparing (24) with (13), which is equal to the throughput of multiuser downlink MIMO-OFDM-TDMA scheduling, $R_{\text{cont.VR,TDMA}}$, we conclude that the opportunistic scheduling enhance the throughput by

$$R_{\text{cont.VR,OPP}} - R_{\text{cont.VR,TDMA}} = \mathrm{E}(\log\eta^*) - n\mathrm{E}(\log\lambda).$$

This enhancement is a function of number of users, U, as illustrated in Fig. 3, where $n = 2$. This observation suggests that opportunistic scheduling does not enhance the pre-log factor. Instead, it increases a constant rate independent of the SNR. Therefore, the throughput gap between opportunistic MIMO-OFDMA and MIMO-OFDM-TDMA becomes narrower as SNR increases. If the channel taps of different users are independent but not identical distributed, the opportunistic scheduling may cause unfair channel access problem. For example, consider a BS is serving two downlink users: user 1 is near the BS and user 2 is far away from it. Due to the shadowing effect, $\sigma_{h_1}^2 > \sigma_{h_2}^2$. According to (10), the eigenvalues of user 1 has better opportunity to be greater than those of user 2. Thus, opportunistic scheduling will allocate most subcarriers to transmit user 1's data. When the difference between $\sigma_{h_1}^2$ and $\sigma_{h_2}^2$ becomes larger, the unfair resource allocation be more severe. This "near-far" effect should be avoided and OFDM-TDMA scheduling is a better choice in this special case.

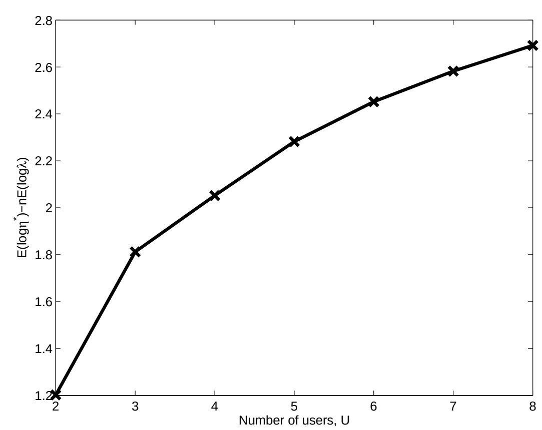

Fig. 3. Throughput enhancement of 2×2 OFDMA with multiuser scheduling over OFDM-TDMA scheduling as a function of number of users, U.

VI. Simulation Results

In first of simulation, we set $N = 64$, $L = 4$ and $n_t = n_r = 4$ with a target BER at 10^{-3}. The receiver does not have the knowledge of CSI so that we reserve $n_r \times L = 16$ pilots from a total of $N = 64$ subcarriers for the training and tracking purpose. Fig. 4 shows that the VRVP scheme performs the best among three schemes since it has two degrees of freedom in this optimization problem. The VR scheme performs the worst. However, the performance gaps narrows as the SNR value becomes higher. The VP scheme performs almost the same as the VRVP scheme in the low SNR region but the performance gap increases with SNR. It is due to the rate loss from the constant constellation constraint. By taking the implementation complexity into account, it is better to choose the VR scheme in the low SNR region and the VP scheme in the high SNR region. We see that both VRVP and VP schemes can achieve noncoherent capacity at any SNR while the VR scheme can only achieve the capacity in the high SNR region. It implies that power allocation is more useful to enhance throughout than rate adaption when the power is limited.

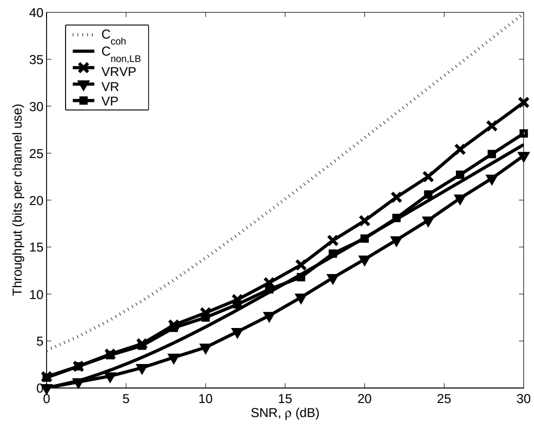

Fig. 4. Average rate for MIMO-OFDM systems of VRVP, VR and VP.

In second part of simulation, we compare the multiuser downlink throughput of MIMO-OFDMA employed with opportunistic scheduling with the throughput of MIMO-OFDM employed with TDMA scheduling. VR schemes is adopted and parameters are set as $M = 1$, $\sigma_h^2 = 1$, $N = 64$, $L = 4$ and $n_t = n_r = n^* = 2$ with a target BER at 10^{-3}. The behavior of the throughput ratio $\frac{R_{\text{OFDMA}}}{R_{\text{TDMA}}}$ as function of number of users, U, and transmit SNR, ρ is shown in Figure 5. We observe that the ratio is decreasing as SNR increasing. For example, the ratio is 1.8 at $\rho = 0$dB and 1.2 at $\rho = 10$dB when 2 users share the downlink. It suggests that the OFDMA with opportunistic scheduling creates a"multiuser diversity" which provides a large gain in low SNR region. However, it does not improve much in middle to high SNR region.

VII. Conclusion

The noncoherent capacity of MIMO-OFDM systems was first derived in this work. Then, an adaptive BIC-MIMO-OFDM scheme that exploit transmitter beamforming and variable-rate-variable-power QAM was proposed to enhance the throughput in a time-varying environment. The performance of adaptive VRVP, VR and VP schemes was analyzed and evaluated via computer simulation. Simulation results also suggested that power allocation is a better choice to enhance throughput than rate adaption when the total power is limited. Theoretical throughput of VR scheme has been proved to have the same pre-log factor as the lower bound of noncoherent capacity. It has been proved that VR scheme has the same diversity-multiplexing tradeoff behavior as MIMO systems employed with random Gaussian codewords. Throughput of

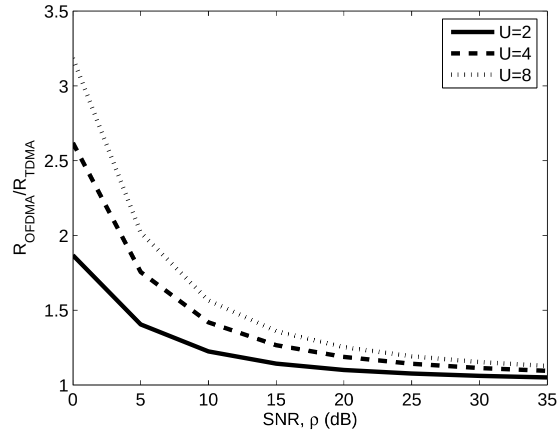

Fig. 5. Throughput ratio of OFDMA and TDMA.

multiuser downlink has been analyzed and OFDMA with opportunistic scheduling outperforms OFDM-TDMA scheduling with a constant enhancement dependent on number of users and number of antennas but independent of SNR.

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