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Joint MIMO Channel Tracking and Symbol Detection with EM Algorithm and Soft Decoding

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***Abstract*— An expectation maximization (EM) algorithm for joint channel tracking and symbol detection in a multi-input multi-output (MIMO) time-varying frequency-selective fading environment is proposed in this research. Based on the recursive EM procedure in conjunction with soft decoding, we develop an iterative algorithm that performs the minimum mean squared error (MMSE) channel estimation and the maximum *a posterior* (MAP) probability symbol detection jointly. Two soft decoders are examined; namely, the BCJR algorithm and the soft sphere decoder. The performance of the proposed algorithm is evaluated via simulation and compared with that of Kalman filtering with hard decision feedback. It is demonstrated by numerical simulation that the proposed algorithm has robust performance in the presence of a severe channel model mismatch.**

I. INTRODUCTION

Space-time coding offers a bandwidth- and power-efficient solution to wireless communications. It exploits multiple transmit and multiple receive antennas to combat fading channels [1]. Although the code design in the original work of Tarokh *et al.* [1] focused primarily on quasi-static flat fading channels, there have been extensions to address more practical models such as time-varying frequency-selective fading channels in recent years [2]. If the channel variation within a transmission block is substantially large, it is important to develop an effective channel tracking scheme to make symbol detection more reliable. This motivates our research on joint channel tracking and symbol detection for such channels.

There have been several schemes proposed for joint maximum likelihood sequence detection (MLSD) and maximum *a posterior* (MAP) or minimum mean squared error (MMSE) channel estimation. Most of these algorithms can be shown to be special applications of the Expectation Maximization (EM) algorithm [3]. Among them, the most well known example is the per-survivor processing (PSP) technique [4]. A drawback of the PSP technique is that its complexity increases exponentially with the channel memory length in time-varying fading channels. A unified structure was proposed in [5] for joint channel tracking and symbol detection in multipath fading channels based on the recursive EM algorithm. Furthermore, in conjunction with the BCJR algorithm, the recursive procedure was presented as a message-passing scheme on a cyclic graph. Here, we extend this result in [5] to multiple-input-multiple-output (MIMO) channels.

The List Sphere Decoder (LSD) was proposed in [6] to approximate the soft information using the closest points in a list found by the sphere decoding algorithm [7]. Together with soft outer-code decoding, it can nearly achieve the capacity of MIMO channels [6], [8]. The result coincides with the well-known turbo detection principle [9]. Furthermore, the maximum likelihood sphere decoding (MLSD) over channels with memory was converted to the closest vector problem from the lattice viewpoint in [10]. In this work, we do not take outer codes into account, but study the performance of LSD in the presence of imperfect channel information. We develop a sliding window soft sphere decoder with soft interference cancelation and compare its performance with that of the BCJR algorithm.

The rest of this paper is organized as follow. A general MIMO system over time-varying frequency-selective fading channels is discussed in Sec. II. We separate the space-time channel matrix into two parts according to their temporal characteristics: the time-invariant angle-delay response and the time-varying multipath gain. An efficient way to estimate the angle-delay pattern was given in [11], and it can be obtained in the training phase in the very beginning of transmission. A joint multipath gain tracking and symbol detection scheme using the EM algorithm combined with soft decoding is proposed in Sec. III. The performance of the proposed scheme is evaluated and contrasted with Kalman filter with hard decision feedback in Sec. IV. Concluding remarks are given in Sec. V.

II. CHANNEL MODEL

Consider a MIMO system with L_t-input, L_r-output antennas over time-varying frequency-selective channels. Then, the baseband multipath channel between transmitter k and the receiver of the base station can be modeled as a single-input multiple-output channel with the following vector impulse response:

$$\underline{h}(t) = \sum_{l=1}^{L_p} \mathbf{a}(\phi_{kl}) h_{kl}(t) \delta(t - \tau_{kl}), \tag{1}$$

where L_p is the number of paths in each transmitter's channel, $h_{kl}(t)$ and τ_{kl} are the complex fading gain and the delay of the lth path of the kth transmitter, respectively, and $\mathbf{a}(\phi_{kl})$ is the corresponding array response vector determined by the array geometry and the angle of arrival ϕ_{kl}. At each receive antenna, the baseband received signal is passed through the matched filter and sampled at the symbol rate. The temporal support of the channel is assumed to be $[0, L_c T]$. The discrete-time model can be written as

$$\mathbf{y}_m = \mathbf{H}_m \mathbf{x}_m + \mathbf{n}_m, \tag{2}$$

where $\mathbf{y}_m = [y_1(m), \cdots, y_{L_r}(m)]^T$ is the $L_r \times 1$ vector collecting the outputs from all receive antennas at time m; $\mathbf{x}_m = [x_1(m), \cdots, x_{L_t}(m), \cdots, x_1(m-L_c+1), \cdots, x_{L_t}(m-L_c+1)]^T$ is the $L_t L_c \times 1$ transmitted vector; $\mathbf{n}_m$ is the $L_r \times 1$ zero-mean complex colored Gaussian noise vector with covariance $\mathbf{C}$, and the space-time channel matrix $\mathbf{H}_m$ can be decomposed as [11]

$$\begin{aligned}\mathbf{H}_m &= \left[\mathbf{a}(\phi_{11}) \cdots \mathbf{a}(\phi_{L_t L_p})\right] \begin{bmatrix} h_{11}(m) & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & h_{L_t L_p}(m) \end{bmatrix} \begin{bmatrix} \mathbf{g}^T(\tau_{11}) \\ \vdots \\ \mathbf{g}^T(\tau_{L_t L_p}) \end{bmatrix} \\ &= \mathbf{A}(\underline{\phi}) \mathrm{diag}(\mathbf{h}_m) \mathbf{G}^T(\underline{\tau}).\end{aligned}$$

where $\mathbf{h}_m = [h_{11}(m) \cdots h_{L_t L_p}(m)]^T$ is the multipath CSI vector at time m and $\mathbf{g}(\tau_{ij})$ is the $L_t L_c \times 1$ vector obtained from a zero vector with the $((k-1) \times L_t + i)$-th element replaced by the k-th element of sampled delay waveform vector of the convolution of the transmitted pulse and the matched filter with respect to the jth-path of the i-th transmitter.

For time-varying channels, the angle of arrival and the time delay of each path are more stationary than the fading channel gain. Thus, we assume that the spatial matrix $\mathbf{A}(\underline{\phi})$ and the temporal matrix $\mathbf{G}(\underline{\tau})$ are time-invariant during M symbols while the channel gain $\mathbf{h}_m$ changes symbol by symbol. We can represent the time variation of the MIMO channel by rearranging the vector form as

$$\begin{aligned}\mathbf{y}_m &= (\mathbf{x}_m^T \otimes \mathbf{I}_{L_r}) \mathrm{vec}(\mathbf{H}_m) + \mathbf{n}_m \\ &= (\mathbf{x}_m^T \otimes \mathbf{I}_{L_r})(\mathbf{G}(\underline{\tau}) \odot \mathbf{A}(\underline{\phi}))\mathbf{h}_m + \mathbf{n}_m \\ &= \mathbf{T}_m \mathbf{h}_m + \mathbf{n}_m,\end{aligned} \tag{3}$$

where $\mathrm{vec}(\cdot)$ is the operator of stacking columns of a matrix into a vector and $\otimes$ and $\odot$ denote the Kronecker and the column-wise Kronecker products, respectively. We assume that the multipath fading gain $h_{kl}(m)$ satisfies the wide-sense-stationary-uncorrelated-scattering (WSSUS) model, which is a zero-mean WSS complex Gaussian process, uncorrelated with any other $h_{k'l'}(m)$. A common way to model the time evolution dynamics of a vector process is to use the autoregressive moving average (ARMA) filter. Let us define $\underline{\mathbf{h}}_m \equiv [\mathbf{h}_m^T, ..., \mathbf{h}_{m-L_h+1}^T]^T$, where L_h is the order of the channel model. The channel dynamics is modeled by a state-space form given by

$$\begin{aligned}\underline{\mathbf{h}}_m &= \mathbf{F}\underline{\mathbf{h}}_{m-1} + \underline{\mathbf{B}}\mathbf{v} \\ &= \begin{bmatrix} \mathbf{F}_1 & \mathbf{F}_2 & \cdots & \mathbf{F}_{L_h} \\ \mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{I} & \mathbf{0} \end{bmatrix} \underline{\mathbf{h}}_{m-1} + \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix} \mathbf{v},\end{aligned} \tag{4}$$

where $\mathbf{F}_i$, $i = 1, \cdots, L_h$ and $\mathbf{B}$ is of dimension $L_t L_p \times L_t L_p$. Due to the WSSUS assumption, $\mathbf{F}_i$ and $\mathbf{B}$ must be diagonal. The diagonal entries can be selected by exploiting the correlation-matching property and solving Yule-Walker's equations in order to match the physical model. Although the approximation accuracy can be improved by increasing the order L_h, the complexity of the tracking algorithm given in the next section will also increase. On the other hand, we will show in Section IV that the proposed algorithm is robust to severely channel model mismatch. Notice that $\mathbf{A}(\underline{\phi})$ and $\mathbf{G}(\underline{\tau})$ have been introduced to account for the spatial and temporal correlations, respectively, and are assumed known from a training phase [11].

III. Proposed Joint Channel Estimation and Symbol Detection Algorithm

A. Recursive EM Algorithm

Let $\mathcal{C}_m$, $\mathcal{I}_m$ and θ_m denote the complete data, incomplete data and channel parameters at time m, respectively. Then, the two steps of the EM algorithm can be written as

1) E step: computing the Kullback-Leibler (K-L) measure

$$Q_m(\theta_m | \hat{\theta}_{m|m}^{(l-1)}) = \mathrm{E}\{\log p(\mathcal{C}_m|\theta_m)|\mathcal{I}_m, \hat{\theta}_{m|m}^{(l-1)}\}. \tag{5}$$

2) M step: maximizing the K-L measure for the new estimate

$$\hat{\theta}_{m|m}^{(l)} = \arg\max_{\theta_m} Q_m(\theta_m | \hat{\theta}_{m|m}^{(l-1)}), \tag{6}$$

where "$|m$" denotes using information up to time m and l is the total number of iterations at each time step. Let $\mathcal{Y}_m = [\mathbf{y}_m^T, \cdots, \mathbf{y}_0^T]^T$ be the received information, $\mathcal{X}_m = \{\mathbf{x}_m, \cdots, \mathbf{x}_0\}$ be the transmitted data up to time m. Let $\mu_{i|m} = \mathrm{E}\{\mathbf{h}_i | \mathcal{Y}_m, \hat{\theta}_{m|m}^{(l-1)}\}$, $\underline{\mu}_{i|m} = \mathrm{E}\{\underline{\mathbf{h}}_i | \mathcal{Y}_m, \hat{\theta}_{m|m}^{(l-1)}\}$, and $\Sigma_{i,j|m}$, $i, j = 0, \cdots, m$, be the cross-covariance matrix between $\mathbf{h}_i$ and $\mathbf{h}_j$. For the convenience of expression, we define $\hbar_m = [\mathbf{h}_m^T, \cdots, \mathbf{h}_0^T]^T$ with conditional mean $\mathcal{U}_{m|m} = [\mu_{m|m}^T, \cdots, \mu_{0|m}^T]^T$ and conditional covariance matrix $\Gamma_{i|m} = [\Sigma_{i-j,i-l}]_{j+1,l+1}$, which is a block matrix with the block at the $(j+1)$th row and $(l+1)th$ column equal to $\Sigma_{i-j,i-l}$. In this problem, the complete and incomplete data at time m are denoted by $\mathcal{C}_m = \{\mathcal{Y}_m, \hbar_m, \mathcal{X}_m\}$ and $\mathcal{I}_m = \{\mathcal{Y}_m\}$, respectively, and $\theta_{m|m} = \{\mathcal{U}_{m|m}, \mathbf{C}\}$. The K-L measure with respect to $\mathcal{U}_{m|m}$ in this case is given by [5]

$$\begin{aligned}&Q_m(\mathcal{U}_{m|m}, \hat{\mathbf{C}}_{|m-1} | \hat{\theta}_{m|m}^{(l-1)}) \\ &= \mathrm{E}\{\log p(\mathcal{Y}_m, \mathcal{X}_m, \hbar_m | \mathcal{U}_{m|m}, \hat{\mathbf{C}}_{|m-1}) | \mathcal{Y}_m, \hat{\theta}_{m|m}^{(l-1)}\} \\ &= \textstyle\sum_{i=1}^m \{-\rho_i(\tilde{\mu}_{i|m}, \mu_{i|m}) - \eta_i(\mu_{i|m}, \bar{\mu}_{i|m}) + c_i\} + d \\ &- [\mu_{0|m} - \mu_{-1}]^H \Sigma_{-1}^{-1} [\mu_{0|m} - \mu_{-1}] - \rho_0(\mu_{0|m}, \tilde{\mu}_{0,m}),\end{aligned} \tag{7}$$

where c_i and d are constant terms, and

$$\begin{aligned}\rho_i(\tilde{\mu}_{i|m}, \mu_{i|m}) &= [\tilde{\mu}_{i|m} - \mu_{i|m}]^H \tilde{\mathbf{C}}_{i|m} [\tilde{\mu}_{i|m} - \mu_{i|m}],\\ \eta_i(\mu_{i|m}, \bar{\mu}_{i|m}) &= [\mu_{i|m} - \bar{\mu}_{i|m}]^H (\mathbf{B}\mathbf{B}^H)^{-1} [\mu_{i|m} - \bar{\mu}_{i|m}],\end{aligned}$$

and $\bar{\mu}_{i|m} = \check{\mathbf{F}}\underline{\mu}_{i-1|m}$, $\check{\mathbf{F}} = [\mathbf{F}_1, \cdots, \mathbf{F}_{L_h}]$ and μ_{-1} and Σ_{-1} are initial value for recursion setup. $\tilde{\mathbf{C}}_{i|m}$ and $\tilde{\mu}_{i|m}$ come from the synthetic system that can be characterized by

$$\begin{aligned}\tilde{\mathbf{C}}_{i|m} &= \mathrm{E}\{\mathbf{T}_i^H \hat{\mathbf{C}}_{|m-1}^{-1} \mathbf{T}_i | \mathcal{Y}_m, \hat{\theta}_{m|m}^{(l-1)}\},\\ \tilde{\mathbf{T}}_{i|m} &= \mathrm{E}\{\mathbf{T}_i^H \hat{\mathbf{C}}_{|m-1}^{-1} | \mathcal{Y}_m, \hat{\theta}_{m|m}^{(l-1)}\},\end{aligned}$$

with the synthetic log likelihood function of $\{\mathbf{h}_i, \hat{\mathbf{C}}_{|m-1}\}$ at time i defined as

$$\begin{aligned}&SLL_i(\mathbf{h}_i, \hat{\mathbf{C}}_{|m-1} | \mathcal{Y}_m, \hat{\theta}_{m|m}^{(l-1)}) = -\log(\pi^{L_r} \det(\hat{\mathbf{C}}_{|m-1}))\\ &\quad -\mathrm{E}\{(\mathbf{y}_i - \mathbf{T}_i \mathbf{h}_i)^H \hat{\mathbf{C}}_{|m-1}^{-1} (\mathbf{y}_i - \mathbf{T}_i \mathbf{h}_i) | \mathcal{Y}_m, \hat{\theta}_{m|m}^{(l-1)}\}\\ &= -[\tilde{\mathbf{C}}_{i|m}^{-1} \tilde{\mathbf{T}}_{i|m} \mathbf{y}_i - \mathbf{h}_i]^H \tilde{\mathbf{C}}_{i|m}^{-1} [\tilde{\mathbf{C}}_{i|m}^{-1} \tilde{\mathbf{T}}_{i|m} \mathbf{y}_i - \mathbf{h}_i] + const.,\end{aligned}$$

which can be maximized (with respect to $\mathbf{h}_i$) by setting $\tilde{\mu}_{i|m} = \tilde{\mathbf{C}}_{i|m}^{-1} \tilde{\mathbf{T}}_{i|m} \mathbf{y}_i$. Therefore, $\tilde{\mu}_{i|m}$ is referred to as the synthetic ML estimate of $\mathbf{h}_i$. Notice that, in (7), except for the constant terms, all other terms are in a Gaussian quadratic form. Thus, maximizing (7) with respect to $\mathcal{U}_{m|m}$ is equivalent to finding

$$\begin{aligned}\hat{\mathcal{U}}_{m|m} &= \arg\max_{\mathcal{U}_{m|m}} P(\mu_{0|m} | \mu_{-1}) P(\tilde{\mu}_{0|m} | \mu_{0|m})\\ &\quad \prod_{i=1}^{m} P(\tilde{\mu}_{i|m} | \mu_{i|m}) P(\mu_{i|m} | \bar{\mu}_{i|m}). \qquad (8)\end{aligned}$$

This is also equivalent to seeking the recursive expression of $\hat{\mathcal{U}}_{i|m}$. The probabilistic model implies that the time evolution characteristic of $\mathcal{U}_{i|m}$ can be modeled by

$$\mu_{i|m} = \check{\mathbf{F}}\underline{\mu}_{i-1|m} + \mathbf{B}\nu_i \text{ and } \tilde{\mu}_{i|m} = \mu_{i|m} + \omega_i,$$

where $\nu_i \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$ and $\omega_i \sim \mathcal{N}(\mathbf{0}, \tilde{\mathbf{C}}_{i|m}^{-1})$. The recursive stochastic MMSE channel estimator is given by a series of Kalman-like equations. They can be written as

$$\begin{aligned}\hat{\mathcal{U}}_{i|m} &= \hat{\mathcal{U}}'_{i|m} + \mathbf{K}_{i|m}(\tilde{\mu}_{i|m} - \mathbf{J}_i^H \hat{\mathcal{U}}'_{i|m}), &(9)\\ \hat{\Gamma}_{i|m} &= \hat{\Gamma}'_{i|m} - \mathbf{K}_{i|m} \mathbf{J}_i^H \hat{\Gamma}'_{i|m}, &(10)\\ \mathbf{K}_{i|m} &= \hat{\Gamma}'_{i|m} \mathbf{J}_i (\tilde{\mathbf{C}}_{i|m}^{-1} + \mathbf{J}_i^H \hat{\Gamma}'_{i|m} \mathbf{J}_i)^{-1}, &(11)\end{aligned}$$

where $\mathbf{J}_i = [\mathbf{I}_{L_t L_p \times L_t L_p}, \mathbf{0}_{L_t L_p \times i L_t L_p}]^H$ and $\hat{\mathcal{U}}'_{i|m}$ and $\hat{\Gamma}'_{i|m}$ are obtained via

$$\begin{aligned}\hat{\mathcal{U}}'_{i|m} &= [\bar{\mu}_{i|m}^T, \mathcal{U}_{i-1|m}^T]^T = [(\check{\mathbf{F}}\underline{\mu}_{i-1|m})^T, \mathcal{U}_{i-1|m}^T]^T,\\ \hat{\Gamma}'_{i|m} &= \begin{bmatrix} \check{\mathbf{F}}\underline{\hat{\Sigma}}_{i-1,i-1|m}\check{\mathbf{F}}^H & \check{\mathbf{F}}\mathbf{V}_{i-1|m}^H \\ \mathbf{V}_{i-1|m}\check{\mathbf{F}}^H & \hat{\Gamma}_{m-1|m} \end{bmatrix} + \mathbf{J}_i \mathbf{B}\mathbf{B}^H \mathbf{J}_i^H,\end{aligned}$$

where $\mathbf{V}_{i-1|m} = \mathrm{E}\{[\mathcal{U}_{i-1|m} - \hat{\mathcal{U}}_{i-1|m}][\underline{\mu}_{i-1|m} - \underline{\hat{\mu}}_{i-1|m}]^H\}$. For every time index i, the estimates from time up to i get updated. The dimension of $\hat{\Gamma}_{i|m}$ also increases by $L_t L_p$ at each time step. However, the dimension of matrix $\tilde{\mathbf{C}}_{i|m}^{-1} + \mathbf{J}_i^H \hat{\Gamma}'_{i|m} \mathbf{J}_i$, which requires inversion, is still $L_t L_p \times L_t L_p$. Although the recursive process is similar to the traditional RLS algorithm or Kalman filtering, the main difference is that a synthetic approach is used to average over the *a posterior* probabilities of $\mathbf{x}$. Therefore, the recursive EM can be viewed as a Kalman-filter like algorithm with soft decision feedback. The sequential update of the MMSE covariance estimator can be approximated by

$$\begin{aligned}\hat{\mathbf{C}}_{|m} &= (1 - \frac{1}{m})\hat{\mathbf{C}}_{|m-1} + \frac{1}{m}\tilde{\mathbf{C}}_m^{MMSE}, &(12)\\ \tilde{\mathbf{C}}_m^{MMSE} &= [\mathbf{y}_m - \hat{\mathbf{T}}_{m|m}\hat{\mu}_{m|m}][\mathbf{y}_m - \hat{\mathbf{T}}_{m|m}\hat{\mu}_{m|m}]^H\\ &\quad - \hat{\mathbf{T}}_{m|m}\hat{\mu}_{m|m}\hat{\mu}_{m|m}^H\hat{\mathbf{T}}_{m|m}^H + \tilde{\mathbf{R}}_{m|m}, &(13)\end{aligned}$$

where $\hat{\mathbf{T}}_{m|m} = \mathrm{E}\{\mathbf{T}_m | \mathcal{Y}_m, \theta_{m|m}^{(l-1)}\}$ and

$$\tilde{\mathbf{R}}_{m|m} = \mathrm{E}\{\mathbf{T}_m\hat{\mu}_{m|m}\hat{\mu}_{m|m}^H\mathbf{T}_m^H | \mathcal{Y}_m; \hat{\theta}_{m|m-1}\}.$$

B. Symbol Detection with Soft Decoder

To implement the recursive EM algorithm, the *a posterior* probability (APP) $p(\mathbf{x}_i | \mathcal{Y}_m, \hat{\theta}_{m|m}^{(l-1)})$ is needed. It can be computed by the soft decoders. Thus, to implement the joint channel estimation and symbol detection algorithm, the soft decoder and the recursive EM work in concert, with the soft decoder providing the metrics for the EM estimator and the EM providing the likelihoods required for the metric updates. The recursive procedure can be represented with a graphical model using the concept of message passing [5].

1) BCJR: The BCJR algorithm [12] is probably the most well known soft decoder that also provides the optimal detection over the inter-symbol interference (ISI) channel. A complete description by a finite state machine at time m would require a trellis diagram with $\mathcal{A}^{(L_c-1)\times L_t} \times m$, where $\mathcal{A}$ denotes the alpha-beta size of the modulation. For large L_c and L_t, a full state BCJR algorithm seems not feasible. A sliding window BCJR algorithm will lead to suboptimal performance with a much lower complexity by neglecting the sequences outside the truncation depth L_d. In order to make detection reliable, we stack all the successive received vectors contributed by the transmitted sequence $[x_1(m) \cdots, x_{L_t}(m - L_d + 1)]^T$ as

$$\mathbf{z}_m = \begin{bmatrix} \mathbf{H}_m & \mathbf{0}_{L_r\times L_t} & \cdots & \mathbf{0}_{L_r\times L_t} \\ \mathbf{0}_{L_r\times L_t} & \mathbf{H}_{m-1} & \cdots & \mathbf{0}_{L_r\times L_t} \\ \vdots & \mathbf{0}_{L_r\times L_t} & \ddots & \vdots \\ \mathbf{0}_{L_r\times 1} & \cdots & \mathbf{0}_{L_r\times 1} & \mathbf{H}_{m-L_d+1} \end{bmatrix} \mathbf{s}_m + \mathbf{w}_m, \qquad (14)$$

where

$$\begin{aligned}\mathbf{z}_m &= [\mathbf{y}_m^T, \cdots, \mathbf{y}_{m-L_d+1}^T]^T,\\ \mathbf{s}_m &= [x_1(m), \cdots, x_{L_t}(m - L_d - L_c + 2)]^T,\\ \mathbf{w}_m &= [\mathbf{n}_m^T, \cdots, \mathbf{n}_{m-L_d+1}^T]^T.\end{aligned}$$

Let $\mathbb{X}_{ijk,\pm 1} = \{\mathbf{s} | b_i^k(j) = \pm 1\}$, where $b_i^k(j)$ is the k-th bit of the symbol $x_i(j)$. Then, the soft information can be derived as

$$\lambda[b_i^k(j)] = \ln\frac{P[b_i^k(j) = 1]}{P[b_i^k(j) = 1]} + \ln\frac{\sum_{\mathbb{X}_{ijk,+1}} p(\mathbf{z}|\mathbf{s})P(\mathbf{s}|b_i^k(j))}{\sum_{\mathbb{X}_{ijk,+1}} p(\mathbf{z}|\mathbf{s})P(\mathbf{s}|b_i^k(j))}. \qquad (15)$$

2) Soft Sphere Decoder (SSD): For simplicity, we assume that $L_d = L_c$, *i.e.* we want to detect sequence $\mathbf{x}_m = [x_1(m), \cdots, x_{L_t}(m), \cdots, x_1(m-L_c+1), \cdots, x_{L_t}(m-L_c+1)]^T$ defined in the tracking model. Based on the soft output information of previous iterations, we can form the soft estimate of symbol $x_i(j)$ by averaging each constellation point weighted by the corresponding APP. For example, if a 4-QAM is used, the soft estimate of the k-bit of symbol $x_i(j)$ can be obtained by

$$\hat{b}_i^k(j) = \tanh(\frac{1}{2}\lambda[b_i^k(j)]). \tag{16}$$

Let $\mathbf{x}_{I,m} = [x_1(m-L_c), \cdots, x_{L_t}(m-2L_c+2)]^T$ be the interference sequences respect to the desired sequence $\mathbf{x}_m$, where subscript $_I$ represents interference. Let the channel states corresponding to the desired sequence $\mathbf{x}_m$ in (14) be $\mathcal{H}_m$ and the channel states corresponding to the interference sequence $\mathbf{x}_{I,m}$ be $\mathcal{H}_{I,m}$. Then, the detection model given by (14) can be simplified as

$$\mathbf{z}_m = \begin{bmatrix}\mathcal{H}_m & \mathcal{H}_{I,m}\end{bmatrix} \begin{bmatrix}\mathbf{x}_m \\ \mathbf{x}_{I,m}\end{bmatrix} + \mathbf{w}_m. \tag{17}$$

At time m, we have the soft estimate sequences $\hat{\mathbf{x}}_{I,m}$. Subtracting $\mathcal{H}_{I,m}\hat{\mathbf{x}}_{I,m}$ from (17), we have

$$\mathbf{q}_m = \mathcal{H}_m \mathbf{x}_m + \mathbf{w}'_m, \tag{18}$$

where $\mathbf{q}_m = \mathbf{z}_m - \mathcal{H}_{I,m}\hat{\mathbf{x}}_{I,m}$ and $\mathbf{w}'_m$ accounts for the estimation error plus noise. It is assumed that $\mathbf{n}'_m$ can be approximated by the Gaussian random vector. Then, its covariance matrix can be sequentially updated in a way similar to (12), and the MLSD problem becomes the closest vector problem that can be efficiently solved by various lattice sphere decoding algorithms. This concept was first introduced in [10]. Here, we modify the interference cancelation to be a soft one so that the sliding window type sphere decoding is conducted without error propagation.

The List Sphere Decoder [6] is adopted to perform the soft information calculation. We choose the Zero-Forcing (ZF) point as the initial center since we do not have reliable covariance information of noise in the beginning. After several iterations, a reliable estimate of the covariance matrix is obtained so that we can perform whitening and choose the MMSE point as the center point. The radius can be selected by [8]

$$R \simeq (\frac{N \times |\det(\mathbb{H}_m)|}{V_s})^{1/s}, \tag{19}$$

where $\mathbb{H}_m = \begin{bmatrix} Re\{\mathcal{H}_m\} & -Im\{\mathcal{H}_m\} \\ Im\{\mathcal{H}_m\} & Re\{\mathcal{H}_m\}\end{bmatrix}$ is the lattice generator matrix [7], V_s is the volume of the sphere with unit radius in $\mathbb{R}^s$, $s = 2 \times L_t \times L_d$ and N is the list size. Based on (18), the lattice sphere decoding algorithm is used to search the N closest vectors to the center point (ZF or MMSE). Every time a new point is found, we add it into the list if the list is not full. If the list is full, we compare its radius with the largest radius in the list and drop the larger one. Finally, we use this list $\mathcal{L}$ and the soft information similar to (15) with $\mathbf{z}$ replaced by $\mathbf{q}$, and the summation set replaced by $\mathbb{X}_{ijk,\pm 1} \cap \mathcal{L}$) to approximate the APPs.

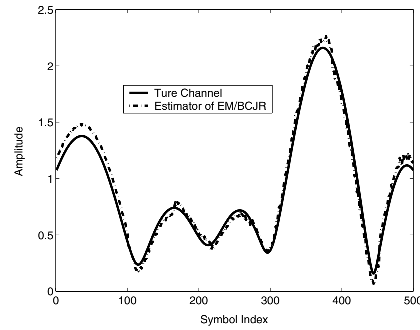

Fig. 1. The channel tracking performance with a normalized Doppler spread $f_D T = 5 \times 10^{-3}$ using EM/BCJR ($E_b/N_0 = 15$dB).

3) Complexity Comparison: The sliding window BCJR algorithm requires $O(\mathcal{A}^{L_c \times L_t})$ operations per symbol and $O(\mathcal{A}^{(L_c-1)\times L_t} \times L_d)$ storage elements. However, as predicted in [10], the overall complexity of the sphere decoder grows as a cubic function of $L_t \times L_d$. It is obvious that the larger the size of $\mathcal{A}$, the more efficient the sphere decoder as compared to the BCJR algorithm.

The major difference between the proposed EM/Soft-decoding approach and the PSP [4] is that, for each survivor path, the corresponding channel estimate is required to update in the PSP case. However, in the proposed algorithm, we only maintain a channel estimate for all paths since the estimate is obtained by averaging over the APPs through a synthetic system.

IV. Simulation Results

The performance of the proposed joint channel tracking and symbol detection algorithm is studied via computer simulation. Signals of two transmitters arrive at an uniform linear array of $L_r = 4$ over a time-varying channel of $L_p = 3$ paths with $L_c = 3$. The fading channel is simulated using an ARMA model of order 31 to match the Jakes' model with a normalized Doppler spread $f_D T = 5 \times 10^{-3}$. The order of the channel model used at the receiver is 3. Thus, there is a mismatch between the model for channel generation and that for channel estimation.

Fig. 1 gives the tracking performance of EM/BCJR for time-varying MIMO-ISI channels. The binary-phase-shift-key (BPSK) modulation is adopted with differential encoding/decoding to compensate for the π radian phase ambiguity in the channel coefficient estimates. The estimation error becomes slightly larger when deep fading occurs. It demonstrates that the third order ARMA model used at the receiver is sufficient in capturing most of the channel dynamics.

Fig. 2 compares the symbol detection performance using perfect CSI with those using estimates obtained from recursive EM and Kalman filtering with hard decision feedback. The reason for the BER deviation between EM tracking and perfect CSI is the mismatch between the ARMA models for channel

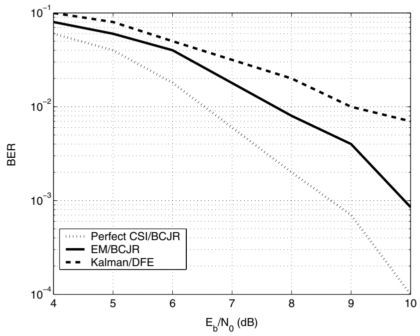

Fig. 2. Performance comparison of the bit error rates (BER) of three schemes.

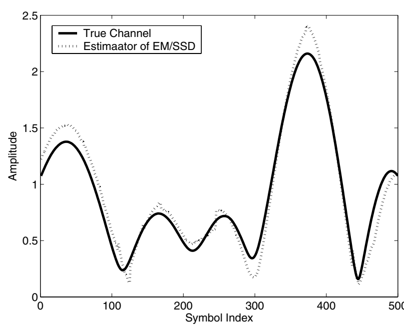

Fig. 3. The channel tracking performance with a normalized Doppler spread $f_D T = 5 \times 10^{-3}$ using EM/SSD ($E_b/N_0 = 15$dB).

generation and estimation. To evaluate the performance of the proposed algorithm, we also consider a receiver employing the Kalman filter with the decision feedback equalizer (DFE). The feedforward and feedback filters of the DFE are chosen to meet the MMSE criteria. The computational complexity for this receiver is much less due to the non-BCJR type detection. However, the price to pay is the inferior performance (about 1.3dB loss for BER at 1×10^{-2}). The relative performance degradation of Kalman filtering with decision feedback to recursive EM is due to the error propagation of the DFE.

Fig. 3 shows the tracking performance of EM with soft sphere decoding over the same channel with a larger signal constellation size, *i.e.* 16-QAM. It is assumed that the channel phase is synchronized with some pilots. Otherwise, we should use a differential-amplitude-phase-encoded (DAPE, also known as STAR) QAM [13] to solve the phase ambiguity. We see that the proposed algorithm works well even with a large input signal constellation.

Fig. 4 compares the 16-QAM symbol detection performance using perfect CSI/BCJR with perfect CSI/SSD, EM/BCJR, EM/SSD and Kalman/DFE. We see that the performance of symbol detection with BCJR outperforms that with SSD. However, the difference is small. It also shows that the performance of EM/BCJR(SSD) has the same slope with perfect CSI/BCJR(SSD). However, the slope of Kalman filtering with DFE is much smoother and results in a big SNR loss (2dB) at the SER level equal to 10^{-2}.

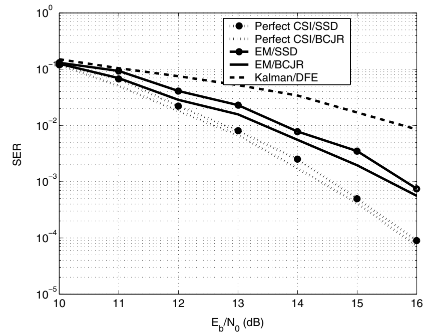

Fig. 4. Performance comparison of the 16-QAM symbol error rates (SER) of five schemes.

V. Conclusion

An iterative approach of joint channel tracking and symbol detection algorithm over time-varying MIMO-ISI channels based on EM algorithm combined with soft decoding was presented. This recursive approach could be viewed as the Kalman filter with soft decision feedback. A soft sphere decoder with soft interference cancelation was proposed to lower the complexity of the BCJR algorithm when large input signal constellation was used. The performance of the proposed algorithm was evaluated via simulations and was showed to be robust to channel model mismatch.

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